ELTR 135 (Operational Amplifiers 2), section 2

Recommended schedule

Day 1

Topics: *Operational amplifier oscillators*
Questions: *1 through 10*
Lab Exercise: *Opamp relaxation oscillator (question 46)*

Day 2

Topics: *Calculus explained through active integrator and differentiator circuits*
Questions: *11 through 20*
Lab Exercise: *Opamp triangle wave generator (question 47)*

Day 3

Topics: *Logarithm review*
Questions: *21 through 35*
Lab Exercise: *Opamp LC resonant oscillator (question 48)*

Day 4

Topics: *Log/antilog circuits* (optional)
Questions: *36 through 45*
Lab Exercise: *Work on project*

Day 5

Topics: *Review*
Lab Exercise: *Work on project*
Show picture(s) of analog computers

Day 6

Exam 2: *includes Oscillator circuit performance assessment*
Project due
Question 49: Sample project grading criteria

Troubleshooting practice problems
Questions: *50 through 59*

General concept practice and challenge problems
Questions: *60 through the end of the worksheet*

Skill standards addressed by this course section

EIA *Raising the Standard; Electronics Technician Skills for Today and Tomorrow,* June 1994

E Technical Skills – Analog Circuits
E.10 Understand principles and operations of operational amplifier circuits.
E.11 Fabricate and demonstrate operational amplifier circuits.
E.12 Troubleshoot and repair operational amplifier circuits.
E.20 Understand principles and operations of sinusoidal and non-sinusoidal oscillator circuits.
E.21 Troubleshoot and repair sinusoidal and non-sinusoidal oscillator circuits.

B Basic and Practical Skills – Communicating on the Job
B.01 Use effective written and other communication skills. *Met by group discussion and completion of labwork.*
B.03 Employ appropriate skills for gathering and retaining information. *Met by research and preparation prior to group discussion.*
B.04 Interpret written, graphic, and oral instructions. *Met by completion of labwork.*
B.06 Use language appropriate to the situation. *Met by group discussion and in explaining completed labwork.*
B.07 Participate in meetings in a positive and constructive manner. *Met by group discussion.*
B.08 Use job-related terminology. *Met by group discussion and in explaining completed labwork.*
B.10 Document work projects, procedures, tests, and equipment failures. *Met by project construction and/or troubleshooting assessments.*
C Basic and Practical Skills – Solving Problems and Critical Thinking
C.01 Identify the problem. *Met by research and preparation prior to group discussion.*
C.03 Identify available solutions and their impact including evaluating credibility of information, and locating information. *Met by research and preparation prior to group discussion.*
C.07 Organize personal workloads. *Met by daily labwork, preparatory research, and project management.*
C.08 Participate in brainstorming sessions to generate new ideas and solve problems. *Met by group discussion.*
D Basic and Practical Skills – Reading
D.01 Read and apply various sources of technical information (e.g. manufacturer literature, codes, and regulations). *Met by research and preparation prior to group discussion.*
E Basic and Practical Skills – Proficiency in Mathematics
E.01 Determine if a solution is reasonable.
E.02 Demonstrate ability to use a simple electronic calculator.
E.05 Solve problems and [sic] make applications involving integers, fractions, decimals, percentages, and ratios using order of operations.
E.06 Translate written and/or verbal statements into mathematical expressions.
E.09 Read scale on measurement device(s) and make interpolations where appropriate. *Met by oscilloscope usage.*
E.12 Interpret and use tables, charts, maps, and/or graphs.
E.13 Identify patterns, note trends, and/or draw conclusions from tables, charts, maps, and/or graphs.
E.15 Simplify and solve algebraic expressions and formulas.
E.16 Select and use formulas appropriately.
E.17 Understand and use scientific notation.
E.18 Use properties of exponents and logarithms.

Common areas of confusion for students

Difficult concept: *Opamp relaxation oscillator circuit.*

This circuit can be difficult to grasp, because there is a tendency to immediately apply one of the "canonical rules" of opamp circuits: that there is negligible voltage between the inverting and noninverting inputs when negative feedback is present. It is wrong to apply this rule here, though, because this circuit's behavior is dominated by *positive* feedback. This positive feedback comes through a plain resistor network, while the negative feedback comes through a "first-order lag" RC time constant network, which means the positive feedback is immediate while the negative feedback is delayed. Because of this, the opamp output swings back and forth between saturated states, and never settles to an equilibrium position. Thus, it does not act like a simple amplifier and there will be substantial voltage between the two opamp inputs.

A better way to consider this circuit is to think of it as a comparator with hysteresis, the inverting input "chasing" the output with a time lag.

Difficult concept: *Rates of change.*

When studying integrator and differentiator circuits, one must think in terms of how fast a variable is changing. This is the first hurdle in calculus: to comprehend what a rate of change is, and it is not obvious. One thing I really like about teaching electronics is that capacitor and inductors naturally exhibit the calculus principles of integration and differentiation (with respect to time), and so provide an excellent context in which the electronics student may explore basic principles of calculus. Integrator and differentiator circuits exploit these properties, so that the output voltage is approximately either the time-integral or time-derivative (respectively) of the input voltage signal.

It is helpful, though, to relate these principles to more ordinary contexts, which is why I often describe rates of change in terms of *velocity* and *acceleration*. Velocity is nothing more than a rate of change of position: how quickly one's position is changing over time. Therefore, if the variable x describes position, then the derivative $\frac{dx}{dt}$ (rate of change of x over time t) must describe velocity. Likewise, acceleration is nothing more than the rate of change of velocity: how quickly velocity changes over time. If the variable v describes velocity, then the derivative $\frac{dv}{dt}$ must describe velocity. Or, since we know that velocity is itself the derivative of position, we could describe acceleration as the *second derivative* of position: $\frac{d^2x}{dt^2}$

**Difficult concept:** *Derivative versus integral.*

The two foundational concepts of calculus are inversely related: *differentiation* and *integration* are flip-sides of the same coin. That is to say, one "un-does" the other. If you can grasp what one of these operations is, then the other is simply the reverse.

One of the better ways to illustrate the inverse nature of these two operations is to consider them in the context of motion analysis, relating *position* (x), velocity (v), and *acceleration* (a). Differentiating with respect to time, the derivative of position is velocity ($v = \frac{dx}{dt}$), and the derivative of velocity is acceleration ($a = \frac{dv}{dt}$). Integrating with respect to time, the integral of acceleration is velocity ($v = \int a \, dt$) and the integral of velocity is position ($x = \int v \, dt$).

Fortunately, electronics provides a ready context in which to understand differentiation and integration. It is very easy to build *differentiator* and *integrator* circuits, which take a voltage signal input and differentiate or integrate (respectively) that signal with respect to time. This means if we have a voltage signal from a velocity sensor measuring the velocity of an object (such as a robotic arm, for example), we may send that signal through a differentiator circuit to obtain a voltage signal representing the robotic arm's acceleration, or we may send the velocity signal through a integrator circuit to obtain a voltage signal representing the robotic arm's position.

3

Question 1

This is a very common opamp oscillator circuit, technically of the *relaxation* type:
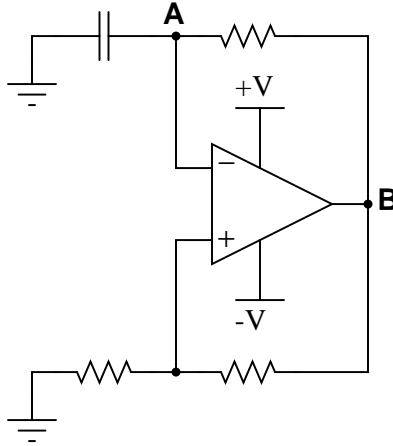
Explain how this circuit works, and what waveforms will be measured at points A and B. Be sure to make reference to *RC time constants* in your explanation.

file 01171

Answer 1

You will measure a sawtooth-like waveform at point A, and a square wave at point B.

Challenge question: explain how you might go about calculating the frequency of such a circuit, based on what you know about RC time constant circuits. Assume that the opamp can swing its output rail-to-rail, for simplicity.

Notes 1

This circuit is best understood by building and testing. If you use large capacitor values and/or a large-value resistor in the capacitor's current path, the oscillation will be slow enough to analyze with a voltmeter rather than an oscilloscope.

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Question 2

A variation on the common opamp relaxation oscillator design is this, which gives it variable duty cycle capability:
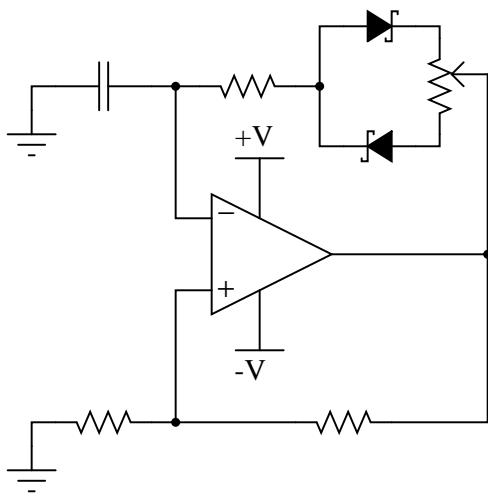
Explain how this circuit works, and which direction the potentiometer wiper must be moved to increase the duty cycle (more time spent with the opamp output saturated at +V and less time spent saturated at -V).

file 02673

Answer 2

Move the wiper *up* to increase the duty cycle.

Notes 2

This circuit is best understood by building and testing. If you use large capacitor values and/or a large-value resistor in the capacitor's current path, the oscillation will be slow enough to analyze with a voltmeter rather than an oscilloscope.

Incidentally, the Schottky diodes are not essential to this circuit's operation, unless the expected frequency is very high. Really, the purpose of the Schottky diodes, with their low forward voltage drops (0.4 volts typical) and minimal charge storage, is to make the opamp's job easier at every reversal of output polarity. Remember that this circuit is not exploiting negative feedback! Essentially, it is a *positive* feedback circuit, and every voltage drop and nonlinearity in the capacitor's current path will have an effect on capacitor charging/discharging.
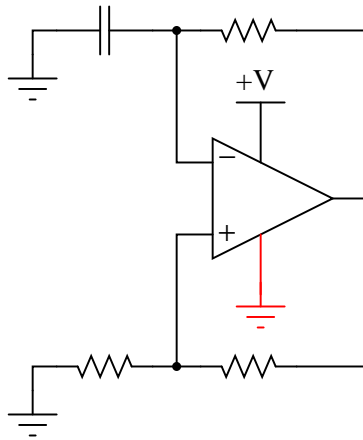
Question 3

Dual, or split, power supplies are very useful in opamp circuits because they allow the output voltage to rise above as well as sink below ground potential, for true AC operation. In some applications, though, it may not be practical or affordable to have a dual power supply to power your opamp circuit. In this event, you need to be able to figure out how to adapt your dual-supply circuit to single-supply operation.

A good example of such a challenge is the familiar opamp relaxation oscillator, shown here:
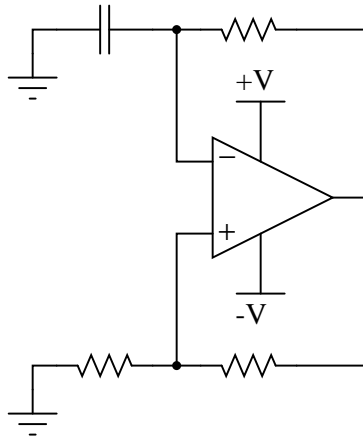
First, determine what would happen if we were to simply eliminate the negative portion of the dual power supply and try to run the circuit on a single supply (+V and Ground only):

Then, modify the schematic so that the circuit will run as well as it did before with the dual supply.
<u>file 02676</u>

6

Answer 3

Here is one solution:
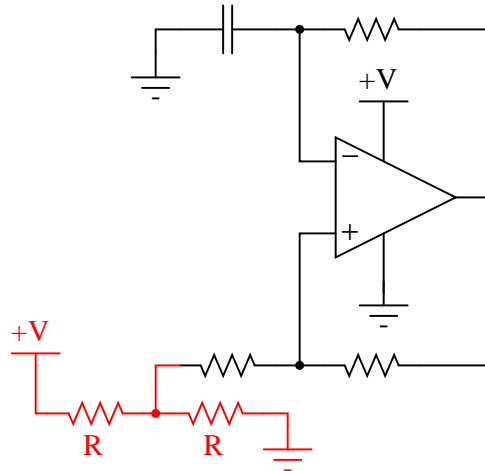
Here is another solution:

$V_{zener} \approx 0.5 \ (V_{supply})$
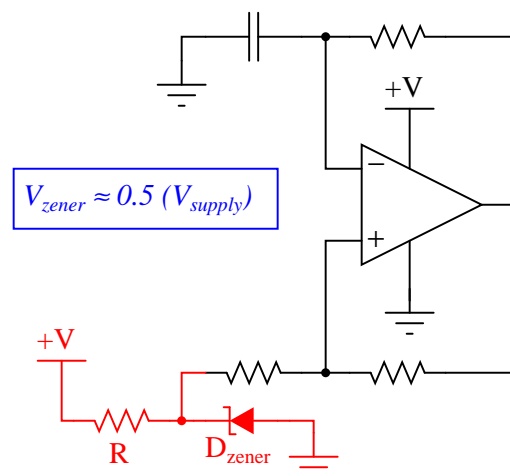
Follow-up question: now you just *know* what I'm going to ask next, don't you? How do these modified circuits function?

Notes 3

Dual power supplies are a luxury in many real-life circumstances, and so your students will need to be able to figure out how to make opamps work in single-supply applications! Work with your students to analyze the function of the suggested solution circuit, to see how it is at once similar and different from its simpler, dual-supply forbear.

Question 4

How many degrees of phase shift must the feedback circuit (the square box in this schematic) introduce to the signal in order for this inverting amplifier circuit to oscillate?

Power source

Feedback
network

Inverting
amplifier
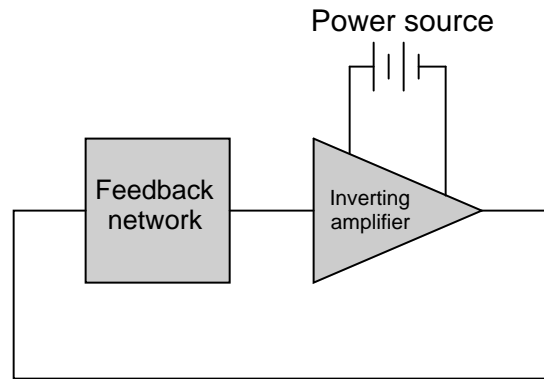
Answer 4

The feedback network in this circuit must provide 180 degrees of phase shift, in order to sustain oscillations.

Notes 4

Ask your students to explain why the feedback network must provide 180 degrees of phase shift to the signal. Ask them to explain how this requirement relates to the need for *regenerative* feedback in an oscillator circuit.

Question 5

How many degrees of phase shift must the feedback circuit (the square box in this schematic) introduce to the signal in order for this noninverting amplifier circuit to oscillate?

Power source

Feedback network

Noninverting amplifier
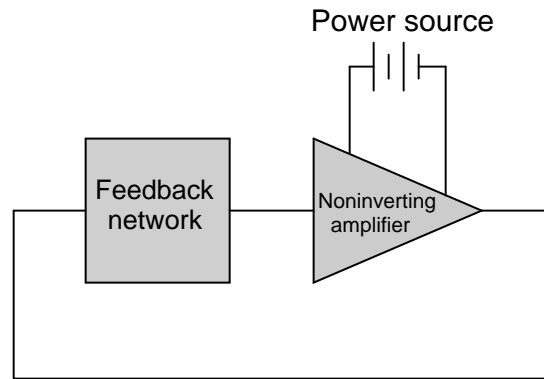
Answer 5

The feedback network in this circuit must provide 360 degrees of phase shift, in order to sustain oscillations.

Notes 5

Ask your students to explain why the feedback network must provide 180 degrees of phase shift to the signal. Ask them to explain how this requirement relates to the need for *regenerative* feedback in an oscillator circuit.

Question 6

Explain what the *Barkhausen criterion* is for an oscillator circuit. How will the oscillator circuit's performance be affected if the Barkhausen criterion falls below 1, or goes much above 1?

file 01211

Answer 6

I'll let you determine exactly what the "Barkhausen" criterion is. If its value is less than 1, the oscillator's output will diminish in amplitude over time. If its value is greater than 1, the oscillator's output will not be sinusoidal!

Notes 6

The question of "What is the Barkhausen criterion" could be answered with a short sentence, memorized verbatim from a textbook. But what I'm looking for here is real comprehension of the subject. Have your students explain to you the reason why oscillation amplitude depends on this factor.

Question 7

Identify what type of oscillator circuit this is, and write an equation describing its operating frequency:
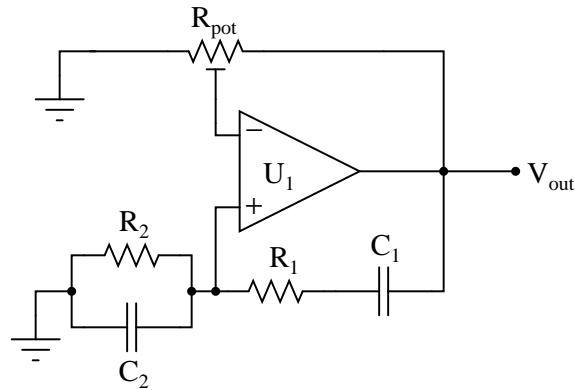
file 02672

Answer 7

This is a *Wien bridge* opamp oscillator, and its operating frequency is determined in the same way as a discrete-transistor Wien bridge oscillator circuit:

$$f = \frac{1}{2\pi RC}$$

Follow-up question: based on your analysis of the circuit, how much phase shift does the Wien bridge circuit introduce into the feedback signal?
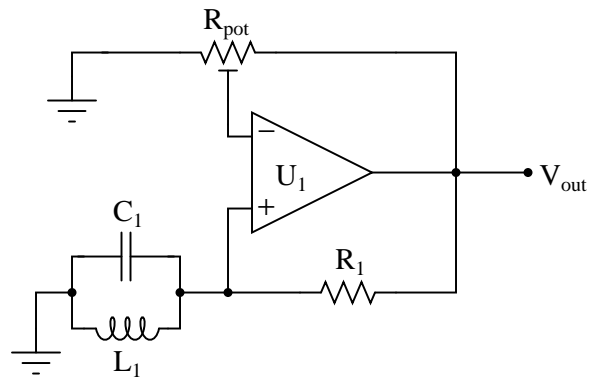
Notes 7

Unlike some discrete transistor oscillator circuits, this Wien bridge is a complete and full Wien bridge, and not a "half-bridge". For an example of a Wien *half*-bridge circuit, look at this (the Wien bridge components shown in a different color):

11
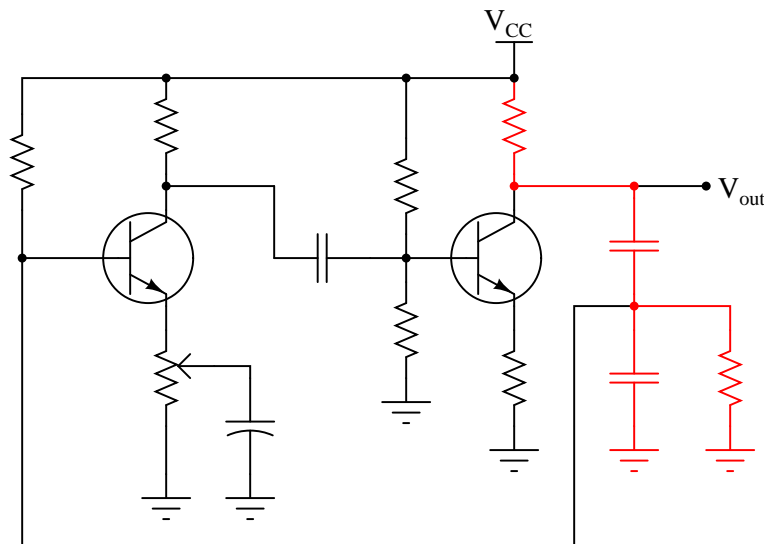
Question 8

Explain the purpose of the tank circuit (L_1 and C_1) in the following oscillator circuit, and write an equation describing its operating frequency:

file 02671

Answer 8

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}}$$

Follow-up question: what do you suppose the purpose of the potentiometer is in this oscillator circuit?
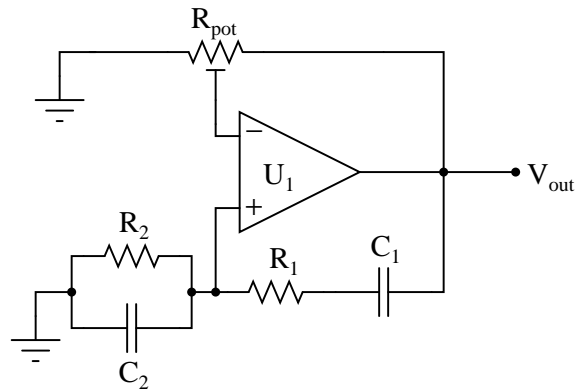
Notes 8

Ask your students to describe the amount of phase shift the tank circuit provides to the feedback signal. Also, ask them to explain how the oscillator circuit's natural frequency may be altered.

Note: potentiometer (voltage gain) adjustment is crucial for obtaining a good-quality sine wave from this type of circuit. If your students decide to build one, they should be aware that some experimentation will be required to get it to output good-quality sine waves!

Question 9

This Wien bridge oscillator circuit is very sensitive to changes in the gain. Note how the potentiometer used in this circuit is the "trimmer" variety, adjustable with a screwdriver rather than by a knob or other hand control:

The reason for this choice in potentiometers is to make accidental changes in circuit gain less probable. If you build this circuit, you will see that tiny changes in this potentiometer's setting make a huge difference in the quality of the output sine wave. A little too much gain, and the sine wave becomes noticeably distorted. Too little gain, and the circuit stops oscillating altogether!

Obviously, it is not good to have such sensitivity to minor changes in any practical circuit expected to reliably perform day after day. One solution to this problem is to add a *limiting network* to the circuit comprised of two diodes and two resistors:

With this network in place, the circuit gain may be adjusted well above the threshold for oscillation (Barkhausen criterion) without exhibiting excessive distortion as it would have without the limiting network. Explain why the limiting network makes this possible.

file 03759

Answer 9

The limiting network attenuates the circuit gain as peak voltage begins to exceed 0.7 volts. This attenuation helps to prevent the opamp from clipping.

Follow-up question: what effect does this "limiting network" have on the purity of the oscillator's output signal spectrum? In other words, does the limiting network increase or decrease the harmonic content of the output waveform?

Notes 9

This circuit is important for students to encounter, as it reveals a very practical limitation of the "textbook" version of the Wien bridge oscillator circuit. It is not enough that a circuit design work in ideal conditions – a practical circuit must be able to tolerate some variance in component values or else it will not operate reliably.

This interesting opamp circuit produces true three-phase sinusoidal voltage waveforms, three of them to be exact:
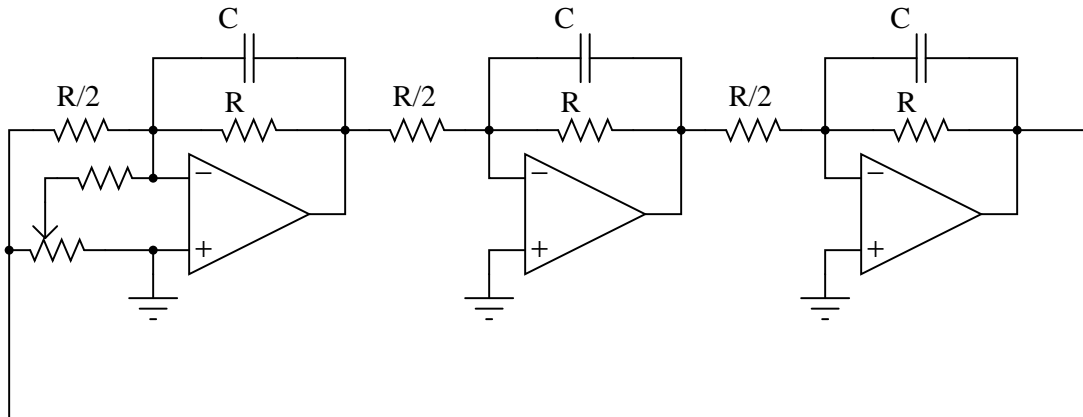
With all the resistors and capacitors, you might have guessed this to be a phase-shift type of oscillator circuit, and you would be correct. Here, each parallel RC network provides 60 degrees of lagging phase shift to combine with the 180 degrees of phase shift inherent to the inverting amplifier configurations, yielding 120 degrees of shift per opamp stage.

Derive a formula solving for the operating frequency of this oscillator circuit, knowing that the impedance of each parallel RC network will have a phase angle of -60°. Also, determine where on this circuit you would obtain the three promised sine waves.

file 02674

Answer 10

$$f = \frac{\sqrt{3}}{2\pi RC}$$

I'll give you a hint on how to solve this problem: the *admittance triangle* for the parallel RC network will have angles of 60°, 30°, and of course 90°:
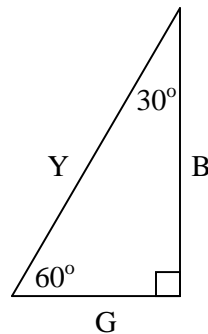
Unlike the multi-stage RC phase shift networks we are accustomed to seeing in discrete transistor phase-shift oscillator circuits, the phase shift networks in this oscillator circuit are much "purer," being effectively isolated from each other by the current gain of each opamp. Here, each RC network provides the exact same amount of phase shift, and is not loaded by the RC network after it. This makes the math nice and easy (comparatively), and a good review of trigonometry!

This circuit came from the pages of one of my favorite opamp books, *Applications Manual for Computing Amplifiers for Modeling, Measuring, Manipulation, and Much Else.* Published by Philbrick Researches Inc. in 1966, it is a wonderfully written tour of "modern" operational amplifier applications and techniques. I only wish (truly) modern texts were written as well as this amazing booklet!

Question 11

How much current will go through the load in this op-amp circuit?
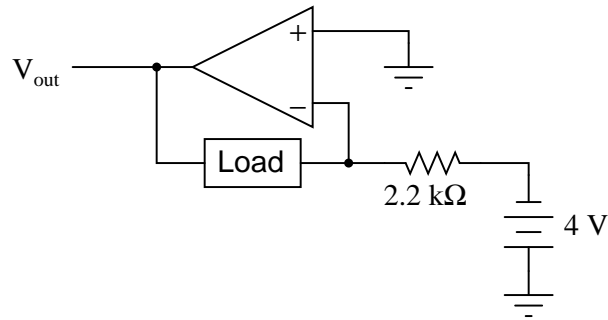
What impact will a change in load resistance have on the operation of this circuit? What will change, if anything, supposing the load resistance were to increase?

file 02694

Answer 11

$I_{load} = 1.818$ mA

If the load changes resistance, there will be no effect on the amount of current through it, although there will be an effect on the output *voltage* of the opamp!

Notes 11

The purpose of this question, besides giving students reason to review a constant-current opamp circuit, is to preview the behavior of an active integrator using the same resistor and input voltage values (question #01008).

Question 12

$\boxed{\int f(x)\, dx \quad \textit{Calculus alert!}}$

How much current will go "through" the capacitor in this op-amp circuit, and what effect does this have on the output voltage?

V$_{\text{out}}$

+

−

10 µF

2.2 kΩ

4 V
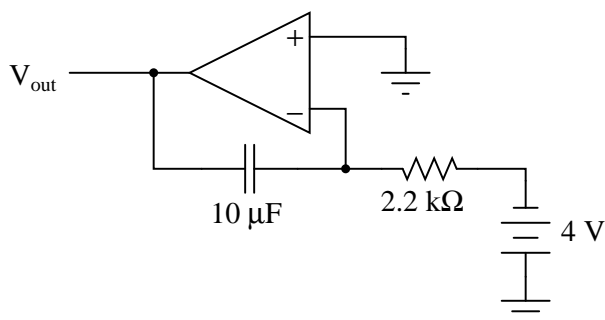
file 01008

Answer 12

$I_C = 1.818$ mA

This circuit is an *integrator*: its output voltage changes over time at a rate proportional to the input voltage magnitude.

Follow-up question: what is the output voltage rate-of-change over time ($\frac{dv}{dt}$) for the circuit shown in the question?

Notes 12

This question is a good review of capacitor theory (relating voltage and current with regard to a capacitor), as well as an introduction to how op-amp circuits can perform calculus functions.

Challenge your students to calculate the output $\frac{dv}{dt}$ without using a calculator!

Question 13

What will the output voltage of this integrator circuit do when the DPDT ("Double-Pole, Double-Throw") switch is flipped back and forth?
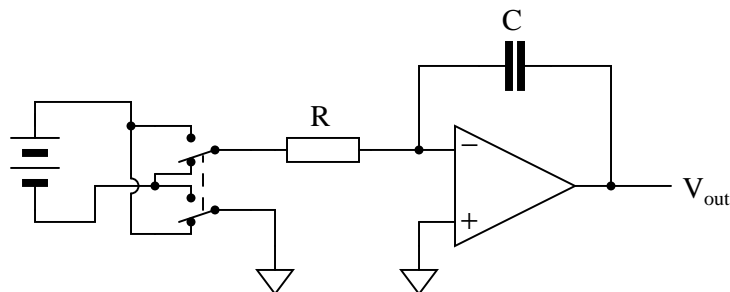

Be as specific as you can in your answer, explaining what happens in the switch's "up" position as well as in its "down" position.

file 02706

Answer 13

With the switch in the "up" position, the opamp output linearly ramps in a negative-going direction over time. With the switch in the "down" position, the opamp output linearly ramps in a positive-going direction over time.

Follow-up question: what do you suppose the output of the following circuit would do over time (assuming the square wave input was true AC, positive and negative)?
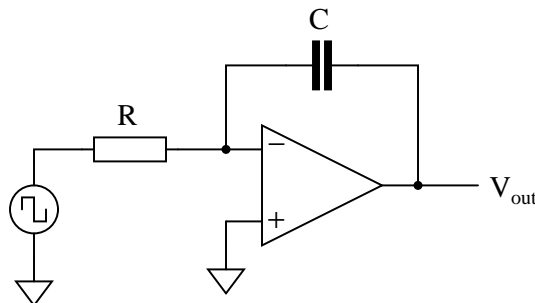
Notes 13

The DPDT switch arrangement may be a bit confusing, but its only purpose is to provide a reversible input voltage polarity. Discuss with your students the directions of all currents in this circuit for both switch positions, and how the opamp output integrates over time for different input voltages.

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Integrator circuits may be understood in terms of their response to DC input signals: if an integrator receives a steady, unchanging DC input voltage signal, it will output a voltage that changes with a steady *rate* over time. The rate of the changing output voltage is directly proportional to the magnitude of the input voltage:

Typical integrator response
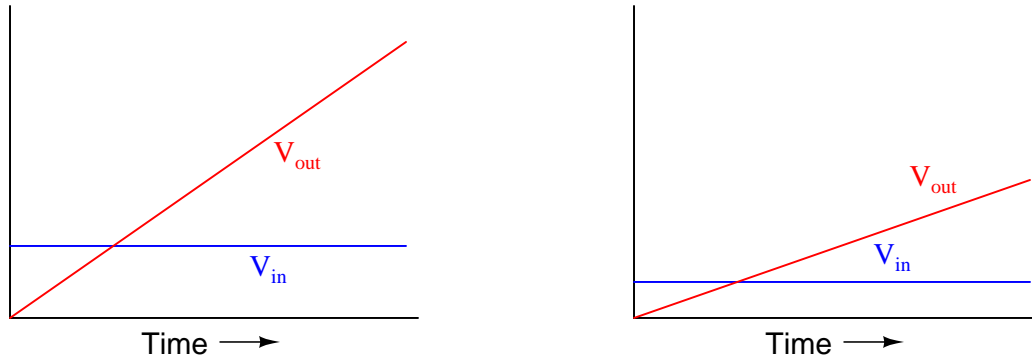
A symbolic way of expressing this input/output relationship is by using the concept of the *derivative* in calculus (a rate of change of one variable compared to another). For an integrator circuit, the rate of output voltage change over time is proportional to the input voltage:

$$\frac{dV_{out}}{dt} \propto V_{in}$$

A more sophisticated way of saying this is, "The time-derivative of output voltage is proportional to the input voltage in an integrator circuit." However, in calculus there is a special symbol used to express this same relationship in reverse terms: expressing the output voltage as a function of the input. For an integrator circuit, this special symbol is called the *integration* symbol, and it looks like an elongated letter "S":

$$V_{out} \propto \int_{0}^{T} V_{in}\,dt$$

Here, we would say that output voltage is proportional to the time-integral of the input voltage, accumulated over a period of time from time=0 to some point in time we call T.

"This is all very interesting," you say, "but what does this have to do with anything in real life?" Well, there are actually a great deal of applications where physical quantities are related to each other by time-derivatives and time-integrals. Take this water tank, for example:

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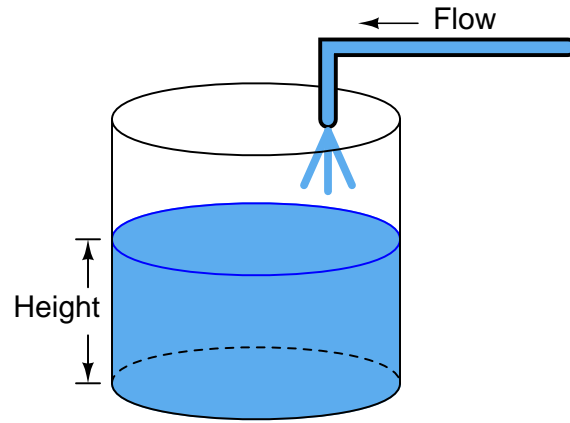
One of these variables (either height H or flow F, I'm not saying yet!) is the time-integral of the other, just as V_{out} is the time-integral of V_{in} in an integrator circuit. What this means is that we could electrically measure one of these two variables in the water tank system (either height or flow) so that it becomes represented as a voltage, then send that voltage signal to an integrator and have the output of the integrator *derive* the other variable in the system without having to measure it!

Your task is to determine which variable in the water tank scenario would have to be measured so we could electronically predict the other variable using an integrator circuit.

file 02695

Answer 14

Flow (F) is the variable we would have to measure, and that the integrator circuit would time-integrate into a height prediction.

Your more alert students will note that the output voltage for a simple integrator circuit is of *inverse* polarity with respect to the input voltage, so the graphs should really look like this:
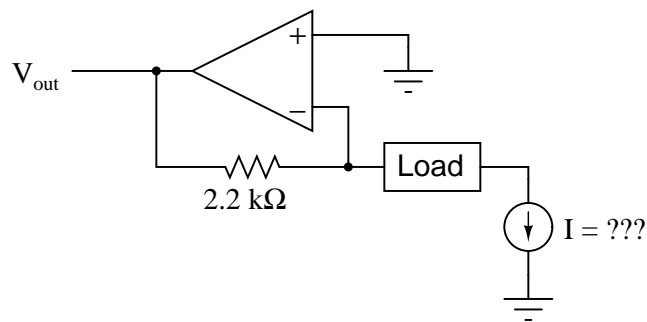
Typical integrator response
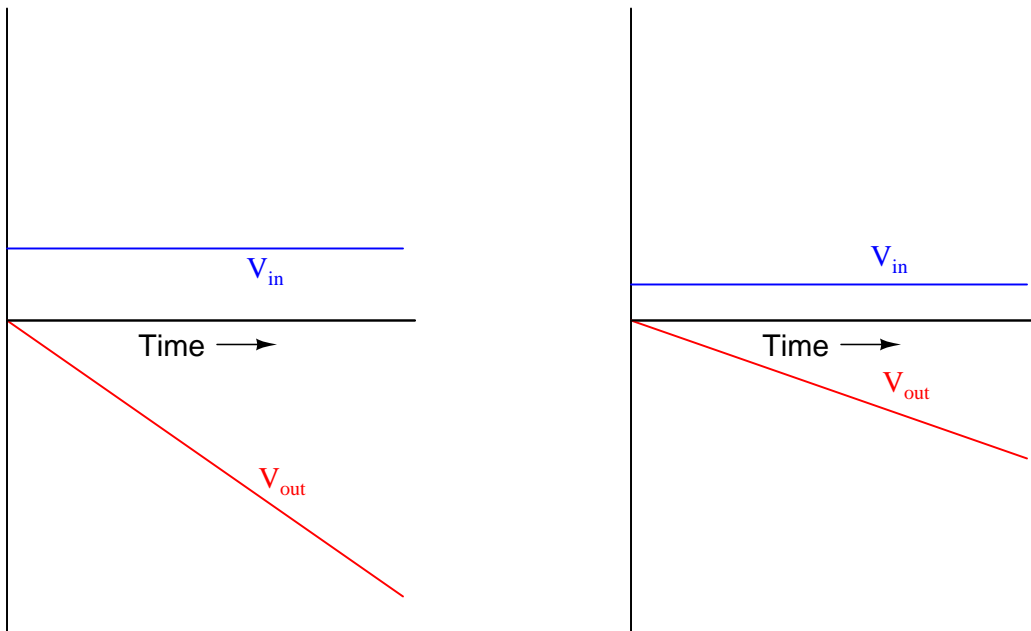
I have chosen to express all variables as positive quantities in order to avoid any unnecessary confusion as students attempt to grasp the concept of time integration.

Question 15

How much current (I) would have to be forced through the resistor in order to generate an output voltage of 5 volts?

Answer 15

$I = 2.273$ mA

Notes 15

The purpose of this question, besides giving students reason to review a current-to-voltage converter opamp circuit, is to preview the behavior of an active differentiator using the same resistor and output voltage values (question #02698).

Question 16

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

How much current (I) would have to be forced through the resistor in order to generate an output voltage of 5 volts?
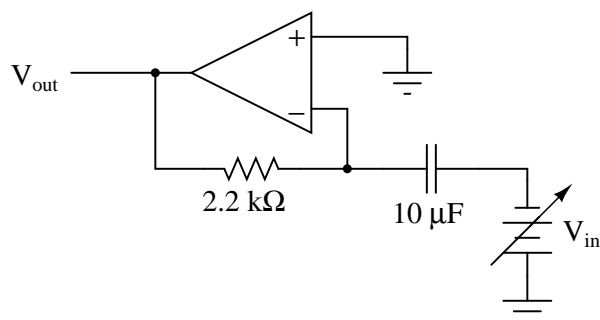
At what *rate* would V_{in} have to increase in order to cause this amount of current to go "through" the capacitor, and thereby cause 5 volts to appear at the V_{out} terminal? What does this tell us about the behavior of this circuit?

file 02698

Answer 16

$I = 2.273$ mA $\qquad\qquad \frac{dV_{in}}{dt} = 227.3$ volts/second

The fact that this circuit outputs a voltage proportional to the *rate of change* over time of the input voltage indicates that it is a *differentiator*.

Notes 16

This question is a good review of capacitor theory (relating voltage and current with regard to a capacitor), as well as an introduction to how op-amp circuits can perform calculus functions.

Question 17

$\boxed{\int f(x)\,dx \quad \text{Calculus alert!}}$

If an object moves in a straight line, such as an automobile traveling down a straight road, there are three common measurements we may apply to it: *position* (x), *velocity* (v), and *acceleration* (a). Position, of course, is nothing more than a measure of how far the object has traveled from its starting point. Velocity is a measure of how *fast* its position is changing over time. Acceleration is a measure of how fast the velocity is changing over time.

These three measurements are excellent illustrations of calculus in action. Whenever we speak of "rates of change," we are really referring to what mathematicians call *derivatives*. Thus, when we say that velocity (v) is a measure of how fast the object's position (x) is changing over time, what we are really saying is that velocity is the "time-derivative" of position. Symbolically, we would express this using the following notation:

$$v = \frac{dx}{dt}$$

Likewise, if acceleration (a) is a measure of how fast the object's velocity (v) is changing over time, we could use the same notation and say that acceleration is the time-derivative of velocity:

$$a = \frac{dv}{dt}$$

Since it took two differentiations to get from position to acceleration, we could also say that acceleration is the *second* time-derivative of position:

$$a = \frac{d^2x}{dt^2}$$

"What has this got to do with electronics," you ask? Quite a bit! Suppose we were to measure the velocity of an automobile using a tachogenerator sensor connected to one of the wheels: the faster the wheel turns, the more DC voltage is output by the generator, so that voltage becomes a direct representation of velocity. Now we send this voltage signal to the input of a *differentiator* circuit, which performs the time-differentiation function on that signal. What would the output of this differentiator circuit then represent with respect to the automobile, *position* or *acceleration*? What practical use do you see for such a circuit?

Now suppose we send the same tachogenerator voltage signal (representing the automobile's velocity) to the input of an *integrator* circuit, which performs the time-integration function on that signal (which is the mathematical inverse of differentiation, just as multiplication is the mathematical inverse of division). What would the output of this integrator then represent with respect to the automobile, *position* or *acceleration*? What practical use do you see for such a circuit?

file 02696

Answer 17

The differentiator's output signal would be proportional to the automobile's *acceleration*, while the integrator's output signal would be proportional to the automobile's *position*.

$$a \propto \frac{dv}{dt} \qquad \text{Output of differentiator}$$

$$x \propto \int_0^T v\,dt \qquad \text{Output of integrator}$$

Follow-up question: draw the schematic diagrams for these two circuits (differentiator and integrator).

25

The calculus relationships between position, velocity, and acceleration are fantastic examples of how time-differentiation and time-integration works, primarily because everyone has first-hand, tangible experience with all three. Everyone inherently understands the relationship between distance, velocity, and time, because everyone has had to travel somewhere at some point in their lives. Whenever you as an instructor can help bridge difficult conceptual leaps by appeal to common experience, do so!

Question 18

$\int f(x)\,dx$ *Calculus alert!*

A familiar context in which to apply and understand basic principles of calculus is the motion of an object, in terms of *position* (x), *velocity* (v), and *acceleration* (a). We know that velocity is the time-derivative of position $(v = \frac{dx}{dt})$ and that acceleration is the time-derivative of velocity $(a = \frac{dv}{dt})$. Another way of saying this is that velocity is the rate of position change over time, and that acceleration is the rate of velocity change over time.

It is easy to construct circuits which input a voltage signal and output either the *time-derivative* or the *time-integral* (the opposite of the derivative) of that input signal. We call these circuits "differentiators" and "integrators," respectively.
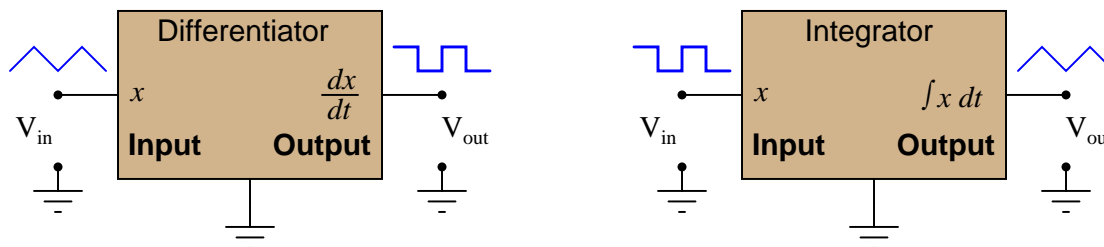
Integrator and differentiator circuits are highly useful for motion signal processing, because they allow us to take voltage signals from motion sensors and convert them into signals representing other motion variables. For each of the following cases, determine whether we would need to use an integrator circuit or a differentiator circuit to convert the first type of motion signal into the second:

- Converting velocity signal to position signal: (*integrator* or *differentiator?*)
- Converting acceleration signal to velocity signal: (*integrator* or *differentiator?*)
- Converting position signal to velocity signal: (*integrator* or *differentiator?*)
- Converting velocity signal to acceleration signal: (*integrator* or *differentiator?*)
- Converting acceleration signal to position signal: (*integrator* or *differentiator?*)

Also, draw the schematic diagrams for these two different circuits.
file 02701

Answer 18

- Converting velocity signal to position signal: (**integrator**)
- Converting acceleration signal to velocity signal: (**integrator**)
- Converting position signal to velocity signal: (**differentiator**)
- Converting velocity signal to acceleration signal: (**differentiator**)
- Converting acceleration signal to position signal: (**two integrators!**)

I'll let you figure out the schematic diagrams on your own!

27

The purpose of this question is to have students apply the concepts of time-integration and time-differentiation to the variables associated with moving objects. I like to use the context of moving objects to teach basic calculus concepts because of its everyday familiarity: anyone who has ever driven a car knows what position, velocity, and acceleration are, and the differences between them.

One way I like to think of these three variables is as a verbal sequence:

Differentiation ➡️

When we change **position** we create **velocity**.

When we change **velocity** we create **acceleration**.

⬅️ *Integration*

Arranged as shown, differentiation is the process of stepping to the right (measuring the *rate of change* of the previous variable). Integration, then, is simply the process of stepping to the left.

Ask your students to come to the front of the class and draw their integrator and differentiator circuits. Then, ask the whole class to think of some scenarios where these circuits would be used in the same manner suggested by the question: motion signal processing. Having them explain how their schematic-drawn circuits would work in such scenarios will do much to strengthen their grasp on the concept of practical integration and differentiation.

Question 19

$\int f(x)\, dx$ *Calculus alert!*

You are part of a team building a rocket to carry research instruments into the high atmosphere. One of the variables needed by the on-board flight-control computer is velocity, so it can throttle engine power and achieve maximum fuel efficiency. The problem is, none of the electronic sensors on board the rocket has the ability to directly measure velocity. What is available is an *altimeter*, which infers the rocket's altitude (it position away from ground) by measuring ambient air pressure; and also an *accelerometer*, which infers acceleration (rate-of-change of velocity) by measuring the inertial force exerted by a small mass.

The lack of a "speedometer" for the rocket may have been an engineering design oversight, but it is still your responsibility as a development technician to figure out a workable solution to the dilemma. How do you propose we obtain the electronic velocity measurement the rocket's flight-control computer needs?

file 02702

Answer 19

One possible solution is to use an electronic *integrator* circuit to derive a velocity measurement from the accelerometer's signal. However, this is not the only possible solution!

Notes 19

This question simply puts students' comprehension of basic calculus concepts (and their implementation in electronic circuitry) to a practical test.

Question 20

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

A *Rogowski Coil* is essentially an air-core current transformer that may be used to measure DC currents as well as AC currents. Like all current transformers, it measures the current going through whatever conductor(s) it encircles.

Normally transformers are considered AC-only devices, because electromagnetic induction requires a *changing* magnetic field $(\frac{d\phi}{dt})$ to induce voltage in a conductor. The same is true for a Rogowski coil: it produces a voltage only when there is a change in the measured current. However, we may measure any current (DC or AC) using a Rogowski coil if its output signal feeds into an integrator circuit as shown:

*Rogowski coil
(air-core current transformer)*
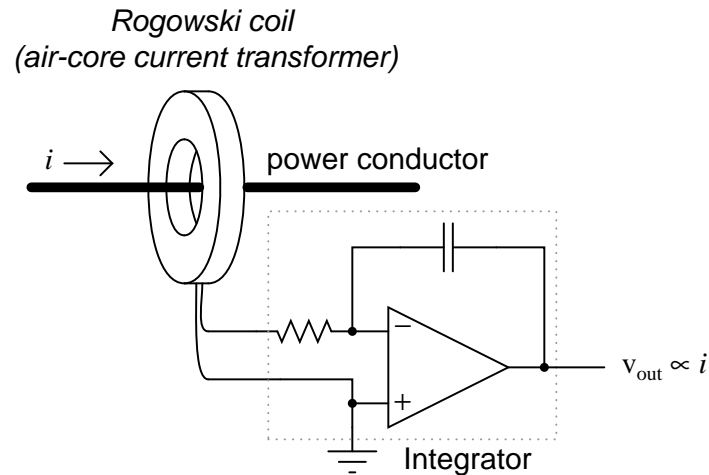
Connected as such, the output of the integrator circuit will be a direct representation of the amount of current going through the wire.

Explain why an integrator circuit is necessary to condition the Rogowski coil's output so that output voltage truly represents conductor current.

file 01009

Answer 20

The coil produces a voltage proportional to the conductor current's rate of change over time ($v_{coil} = M\frac{di}{dt}$). The integrator circuit produces an output voltage changing at a rate proportional to the input voltage magnitude ($\frac{dv_{out}}{dt} \propto v_{in}$). Substituting algebraically:

$$\frac{dv_{out}}{dt} = M\frac{di}{dt}$$

Review question: Rogowski coils are rated in terms of their *mutual inductance* (M). Define what "mutual inductance" is, and why this is an appropriate parameter to specify for a Rogowski coil.

Follow-up question: the operation of a Rogowski coil (and the integrator circuit) is probably easiest to comprehend if one imagines the measured current starting at 0 amps and linearly increasing over time. Qualitatively explain what the coil's output would be in this scenario and then what the integrator's output would be.

Challenge question: the integrator circuit shown here is an "active" integrator rather than a "passive" integrator. That is, it contains an amplifier (an "active" device). We could use a passive integrator circuit instead to condition the output signal of the Rogowski coil, but only if the measured current is purely AC. A passive integrator circuit would be insufficient for the task if we tried to measure a DC current – only an active integrator would be adequate to measure DC. Explain why.

Notes 20

This question provides a great opportunity to review Faraday's Law of electromagnetic induction, and also to apply simple calculus concepts to a practical problem. The coil's natural function is to *differentiate* the current going through the conductor, producing an output voltage proportional to the current's rate of change over time ($v_{out} \propto \frac{di_{in}}{dt}$). The integrator's function is just the opposite. Discuss with your students how the integrator circuit "undoes" the natural calculus operation inherent to the coil (differentiation).

The subject of Rogowski coils also provides a great opportunity to review what mutual inductance is. Usually introduced at the beginning of lectures on transformers and quickly forgotten, the principle of mutual inductance is at the heart of every Rogowski coil: the coefficient relating instantaneous current change through one conductor to the voltage induced in an *adjacent* conductor (magnetically linked).

$$v_2 = M\frac{di_1}{dt}$$

Unlike the iron-core current transformers (CT's) widely used for AC power system current measurement, Rogowski coils are inherently linear. Being air-core devices, they lack the potential for saturation, hysteresis, and other nonlinearities which may corrupt the measured current signal. This makes Rogowski coils well-suited for high frequency (even RF!) current measurements, as well as measurements of current where there is a strong DC bias current in the conductor. By the way, this DC bias current may be "nulled" simply by re-setting the integrator after the initial DC power-up!

If time permits, this would be an excellent point of departure to other realms of physics, where op-amp signal conditioning circuits can be used to "undo" the calculus functions inherent to certain physical measurements (acceleration vs. velocity vs. position, for example).

The concept of a mathematical *power* is familiar to most students of algebra. For instance, ten to the third power means this:

$$10^3 = 10 \times 10 \times 10 = 1000$$

. . . and eight to the seventh power means this:

$$8^7 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 = 2,097,152$$

Just as subtraction is the inverse function of addition, and division is the inverse function of multiplication (because with inverse functions, one "undoes" the other), there is also an inverse function for a power and we call it the *logarithm*.

Re-write the expression $10^3 = 1000$ so that it uses the same quantities (10, 3, and 1000) in the context of a logarithm instead of a power, just as the subtraction is shown here to be the inverse of addition, and division is shown to be the inverse of multiplication in the following examples:

$$3 + 8 = 11 \qquad (+ \text{ and - are inverse functions}) \qquad 11 - 3 = 8$$

$$2 \times 7 = 14 \qquad (\times \text{ and } \div \text{ are inverse functions}) \qquad 14 \div 2 = 7$$

$$10^3 = 1000 \qquad (\text{powers and logs are inverse functions}) \qquad \log_{10} ??? = ???$$

file 02677

Answer 21

$$10^3 = 1000 \qquad (\text{powers and logs are inverse functions}) \qquad \log_{10} 1000 = 3$$

Notes 21

In my experience, most American students are woefully underprepared for the subject of logarithms when they study with me. Admittedly, logarithms do not see as much use in everyday life as powers do (and that is very little for most people as it is!). Logarithms used to be common fare for secondary school and college students, as they were essential for the operation of a *slide rule*, an elegant mechanical analog computing device popular decades ago.

The purpose of this question is to twofold: to get students to realize what a logarithm is, and also to remind them of the concept of inverse functions, which become very important in analog computational circuits.

Question 22

Given the following mathematical expression, write another one defining a *logarithm* using the same variables:

$$\text{If: } x^y = z \qquad \text{Then: } \log_? ? = ?$$

Answer 22

$$\text{If: } x^y = z \qquad \text{Then: } \log_x z = y$$

Notes 22

Nothing special here. Indeed, the answer to this question may be derived from any algebra textbook.

Question 23

Electronic calculators with logarithm capability have at least two different types of logarithms: *common* logarithm and *natural* logarithm, symbolized as "log" and "ln", respectively. Explain what the difference is between these two types of logarithms.

file 02678

Answer 23

The common logarithm function assumes a "base" value of ten, whereas the natural logarithm assumes a base value of e (Euler's constant).

Follow-up question: what is the approximate value of e? How can you get your calculator to give you the answer (rather than looking it up in a math book?

Notes 23

Some calculators, of course, allow you to extract the logarithm of any number to any base. Here, I simply want students to become familiar with the two logarithm functions available on the most basic scientific calculators.

Note that some calculators will show just enough digits of e to give the false impression that they repeat (ten digits: $e = 2.718281828$). If anyone suggests that e is a (rational) repeating decimal number, correct this misunderstanding by telling them it is irrational just like π.

Question 24

Note the following logarithmic identities, using the "common" (base 10) logarithm:

$$\log 10 = 1$$

$$\log 100 = 2$$

$$\log 1000 = 3$$

$$\log 10000 = 4$$

In the first equation, the numbers 10 and 1 were related together by the log function. In the second equation, the numbers 100 and 2 were related together by the same log function, and so on.

Rewrite the four equations together in such a way that the same numbers are related to each other, but without writing "log". In other words, represent the same mathematical relationships using some mathematical function other than the common logarithm function.

file 02680

Answer 24

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1000$$

$$10^4 = 10000$$

Notes 24

An illustration like this helps students comprehend what the "log" function actually does.

Note the following logarithmic identities, using the "common" (base 10) logarithm:

$$\log 0.1 = -1$$

$$\log 0.01 = -2$$

$$\log 0.001 = -3$$

$$\log 0.0001 = -4$$

In the first equation, the numbers 0.1 and 1 were related together by the log function. In the second equation, the numbers 0.01 and 2 were related together by the same log function, and so on.

Rewrite the four equations together in such a way that the same numbers are related to each other, but without writing "log". In other words, represent the same mathematical relationships using some mathematical function other than the common logarithm function.

file 02681

Answer 25

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

$$10^{-4} = 0.0001$$

Notes 25

An illustration like this helps students comprehend what the "log" function actually does.

Question 26

Examine the following progression of mathematical statements:

$$(10^2)(10^3) = 100000$$

$$10^{2+3} = 100000$$

$$10^5 = 100000$$

What does this pattern indicate? What principle of algebra is illustrated by these three equations? Next, examine this progression of mathematical statements:

$$\log 10^5 = \log 100000 = 5$$

$$\log 10^{2+3} = \log 100000 = 5$$

$$\log 10^2 + \log 10^3 = \log 100000 = 5$$

What does this pattern indicate? What principle of algebra is illustrated by these three equations?
file 02682

Answer 26

First pattern:

The product of two base numbers with different exponents is equal to that base number raised to the power of the exponents' sum.

Second pattern:

The sum of two logarithms is equal to the logarithm of those two numbers' product.

Notes 26

In this question, I want students to begin to see how logarithms relate multiplication to addition, and how powers relate addition to multiplication. This is an initial step to students recognizing logarithms as *transform functions*: a means to transform one type of mathematical problem into a simpler type of mathematical problem.

Question 27

Examine this progression of mathematical statements:

$$(100)(1000) = 100000$$

$$(100)(1000) = 10^5$$

$$\log[(100)(1000)] = \log 10^5$$

$$\log 100 + \log 1000 = \log 10^5$$

$$\log 10^2 + \log 10^3 = \log 10^5$$

$$2 + 3 = 5$$

What began as a multiplication problem ended up as an addition problem, through the application of logarithms. What does this tell you about the utility of logarithms as an arithmetic tool?

file 02683

Answer 27

That logarithms can reduce the complexity of an equation from multiplication, down to addition, indicates its usefulness as a tool to *simplify* arithmetic problems. Specifically, the logarithm of a product is equal to the sum of the logarithms of the two numbers being multiplied.

Notes 27

In mathematics, any procedure that reduces a complex type of problem into a simpler type of problem is called a *transform function*, and logarithms are one of the simplest types of transform functions in existence.

Question 28

Suppose you owned a scientific calculator with two broken buttons: the multiply ($\times$) and divide ($\div$). Demonstrate how you could solve this simple multiplication problem using only logarithms, addition, and antilogarithms (powers):

$$7 \times 5 = ???$$

The answer to this problem was easy enough for you to figure out without a calculator at all, so here are some more practice problems for you to try:

- $23 \times 35 =$
- $781 \times 92 =$
- $19.4 \times 60 =$
- $0.019 \times 2.6 =$

file 02685

Answer 28

Here I will show you the steps to using logarithms to solve the first multiplication problem:

$$7 \times 5 = ???$$

$$7 \times 5 = 10^{\log 7 + \log 5}$$

$$7 \times 5 = 10^{0.8451 + 0.6990}$$

$$7 \times 5 = 10^{1.5441}$$

$$7 \times 5 = 35$$

Since the others are easy enough for you to check (with your non-broken calculator!), I'll leave their solutions in your capable hands.

Notes 28

Incidentally, there is nothing special about the common logarithm to warrant its exclusive use in this problem. We could have just as easily applied the natural logarithm function with the same (final) result:

$$7 \times 5 = ???$$

$$7 \times 5 = e^{\ln 7 + \ln 5}$$

$$7 \times 5 = e^{1.9459 + 1.6094}$$

$$7 \times 5 = e^{3.5553}$$

$$7 \times 5 = 35$$

Question 29

Examine this progression of mathematical statements:

$$\frac{1000}{100} = 10$$

$$\frac{1000}{100} = 10^1$$

$$\log\left(\frac{1000}{100}\right) = \log 10^1$$

$$\log 1000 - \log 100 = \log 10^1$$

$$\log 10^3 - \log 10^2 = \log 10^1$$

$$3 - 2 = 1$$

What began as a division problem ended up as a subtraction problem, through the application of logarithms. What does this tell you about the utility of logarithms as an arithmetic tool?

file 02684

Answer 29

That logarithms can reduce the complexity of an equation from division, down to subtraction, indicates its usefulness as a tool to *simplify* arithmetic problems. Specifically, the logarithm of a quotient is equal to the difference between the logarithms of the two numbers being divided.

Notes 29

In mathematics, any procedure that reduces a complex type of problem into a simpler type of problem is called a *transform function*, and logarithms are one of the simplest types of transform functions in existence.

Question 30

Suppose you owned a scientific calculator with two broken buttons: the multiply ($\times$) and divide ($\div$). Demonstrate how you could solve this simple multiplication problem using only logarithms, addition, and antilogarithms (powers):

$$12 \div 3 = ???$$

The answer to this problem was easy enough for you to figure out without a calculator at all, so here are some more practice problems for you to try:

- $122 \div 35 =$
- $781 \div 92 =$
- $19.4 \div 60 =$
- $3.5 \div 0.21 =$

file 02686

Answer 30

Here I will show you the steps to using logarithms to solve the first multiplication problem:

$$12 \div 3 = ???$$

$$12 \div 3 = 10^{\log 12 - \log 3}$$

$$12 \div 3 = 10^{1.0792 - 0.4771}$$

$$12 \div 3 = 10^{0.6021}$$

$$12 \div 3 = 4$$

Since the others are easy enough for you to check (with your non-broken calculator!), I'll leave their solutions in your capable hands.

Notes 30

Incidentally, there is nothing special about the common logarithm to warrant its exclusive use in this problem. We could have just as easily applied the natural logarithm function with the same (final) result:

$$12 \div 3 = ???$$

$$12 \div 3 = e^{\ln 12 - \ln 3}$$

$$12 \div 3 = e^{2.4849 - 1.0986}$$

$$12 \div 3 = e^{1.3863}$$

$$12 \div 3 = 4$$

Question 31

Examine this progression of mathematical statements:

$$(1000)^2 = 1000000$$

$$(1000)^2 = 10^6$$

$$\log[(1000)^2] = \log 10^6$$

$$(2)(\log 1000) = \log 10^6$$

$$(2)(\log 10^3) = \log 10^6$$

$$(2)(3) = 6$$

What began as an exponential problem ended up as a multiplication problem, through the application of logarithms. What does this tell you about the utility of logarithms as an arithmetic tool?

<u>file 02687</u>

Answer 31

That logarithms can reduce the complexity of an equation from exponentiation, down to multiplication, indicates its usefulness as a tool to *simplify* arithmetic problems. Specifically, the logarithm of a number raised to a power is equal to that power multiplied by the logarithm of the number.

Notes 31

In mathematics, any procedure that reduces a complex type of problem into a simpler type of problem is called a *transform function*, and logarithms are one of the simplest types of transform functions in existence.

Question 32

Suppose you owned a scientific calculator with two broken buttons: the power (y^x) and root ($\sqrt[x]{y}$). Demonstrate how you could solve this simple power problem using only logarithms, multiplication, and antilogarithms (powers):

$$3^4 = ???$$

The answer to this problem was easy enough for you to figure out without a calculator at all, so here are some more practice problems for you to try:

- $25^6 =$
- $564^3 =$
- $0.224^2 =$
- $41^{0.3} =$

file 02690

Answer 32

Here I will show you the steps to using logarithms to solve the first multiplication problem:

$$3^4 = ???$$

$$3^4 = 10^{(4 \log 3)}$$

$$3^4 = 10^{(4)(0.4771)}$$

$$3^4 = 10^{1.9085}$$

$$3^4 = 81$$

Since the others are easy enough for you to check (with your non-broken calculator!), I'll leave their solutions in your capable hands.

Notes 32

Incidentally, there is nothing special about the common logarithm to warrant its exclusive use in this problem. We could have just as easily applied the natural logarithm function with the same (final) result:

$$3^4 = ???$$

$$3^4 = e^{(4 \ln 3)}$$

$$3^4 = e^{(4)(1.0986)}$$

$$3^4 = e^{4.3944}$$

$$3^4 = 81$$

Question 33

Examine this progression of mathematical statements:

$$\sqrt{1000} = 10^{1.5}$$

$$\log \sqrt{1000} = \log \left(10^{1.5}\right)$$

$$\log \left(1000^{\frac{1}{2}}\right) = \log \left(10^{1.5}\right)$$

$$\frac{1}{2}(\log 1000) = \log \left(10^{1.5}\right)$$

$$\frac{1}{2}(\log 10^3) = \log \left(10^{1.5}\right)$$

$$\frac{3}{2}(\log 10) = \log \left(10^{1.5}\right)$$

$$\frac{3}{2}(1) = \log \left(10^{1.5}\right)$$

$$\frac{3}{2} = \log \left(10^{1.5}\right)$$

$$\frac{3}{2} = 1.5$$

What began as a fractional exponent problem ended up as a simple fraction, through the application of logarithms. What does this tell you about the utility of logarithms as an arithmetic tool?

file 02688

Answer 33

That logarithms can reduce the complexity of an equation from fractional exponentiation, down to simple fractions, indicates its usefulness as a tool to *simplify* arithmetic problems. Specifically, the logarithm of a root of a number is equal to the logarithm of that number divided by the root index.

Notes 33

In mathematics, any procedure that reduces a complex type of problem into a simpler type of problem is called a *transform function*, and logarithms are one of the simplest types of transform functions in existence.

Question 34

Suppose you owned a scientific calculator with two broken buttons: the power (y^x) and root ($\sqrt[x]{y}$). Demonstrate how you could solve this simple root problem using only logarithms, division, and antilogarithms (powers):

$$\sqrt[3]{8} = ???$$

The answer to this problem was easy enough for you to figure out without a calculator at all, so here are some more practice problems for you to try:

- $\sqrt[4]{13} =$
- $\sqrt[5]{209} =$
- $\sqrt[2.5]{9935} =$
- $\sqrt[9.2]{0.15} =$

file 02689

Answer 34

Here I will show you the steps to using logarithms to solve the first multiplication problem:

$$\sqrt[3]{8} = ???$$

$$\sqrt[3]{8} = 10^{\left(\frac{1}{3} \log 8\right)}$$

$$\sqrt[3]{8} = 10^{\left(\frac{1}{3}(0.9031)\right)}$$

$$\sqrt[3]{8} = 10^{0.3010}$$

$$\sqrt[3]{8} = 2$$

Since the others are easy enough for you to check (with your non-broken calculator!), I'll leave their solutions in your capable hands.

Notes 34

Incidentally, there is nothing special about the common logarithm to warrant its exclusive use in this problem. We could have just as easily applied the natural logarithm function with the same (final) result:

$$\sqrt[3]{8} = ???$$

$$\sqrt[3]{8} = e^{\left(\frac{1}{3} \ln 8\right)}$$

$$\sqrt[3]{8} = e^{\left(\frac{1}{3}(2.0794)\right)}$$

$$\sqrt[3]{8} = e^{0.6931}$$

$$\sqrt[3]{8} = 2$$

Question 35

You may be wondering why anyone would bother using logarithms to solve arithmetic problems for which we have perfectly good and effective digital electronic calculator functions at our disposal. For example, why would anyone do this:

$$10^{\log 7 + \log 5}$$

. . . when they could just do the following on the same calculator?

$$7 \times 5$$

The quick answer to this very good question is, "when it is more difficult to directly multiply two numbers." The trouble is, most people have a difficult time imagining when it would ever be easier to take two logarithms, add them together, and raise ten to that power than it would be to simply multiply the original two numbers together.

The answer to *that* mystery is found in operational amplifier circuitry. As it turns out, it is much easier to build single opamp circuits that add, subtract, exponentiate, or take logarithms than it is to build one that directly multiplies or divides two quantities (analog voltages) together.

We may think of these opamp functions as "blocks" which may be interconnected to perform composite arithmetic functions:
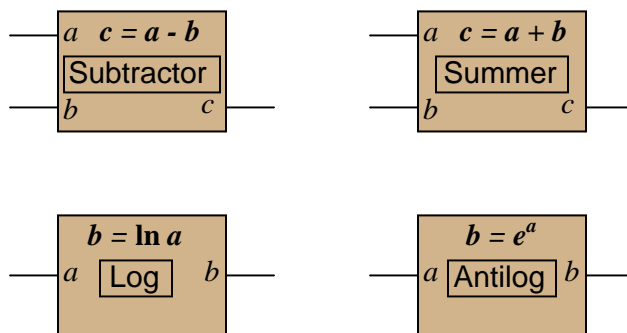
Using this model of specific math-function "blocks," show how the following set of analog math function blocks may be connected together to multiply two analog voltages together:

46
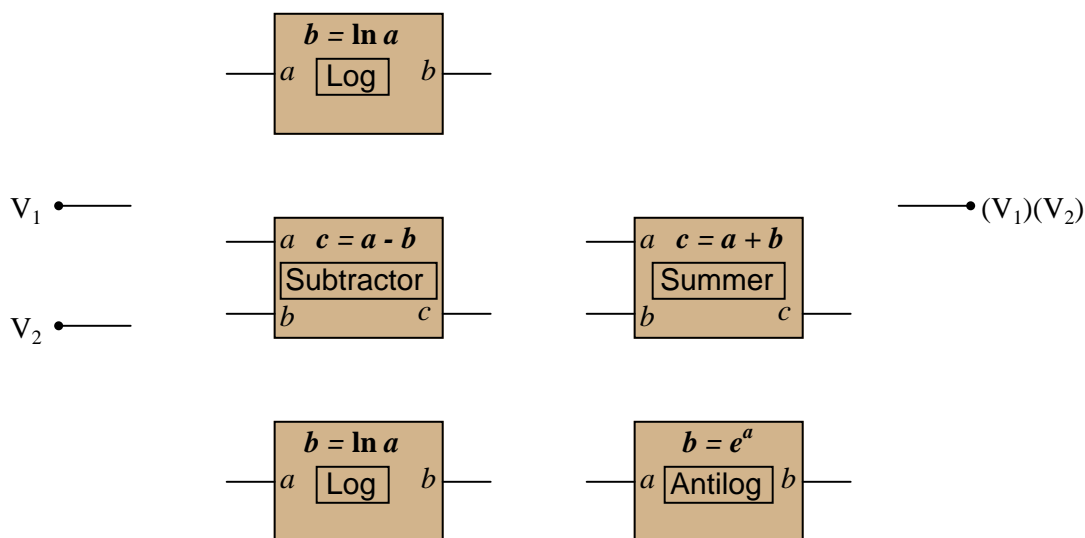
Answer 35
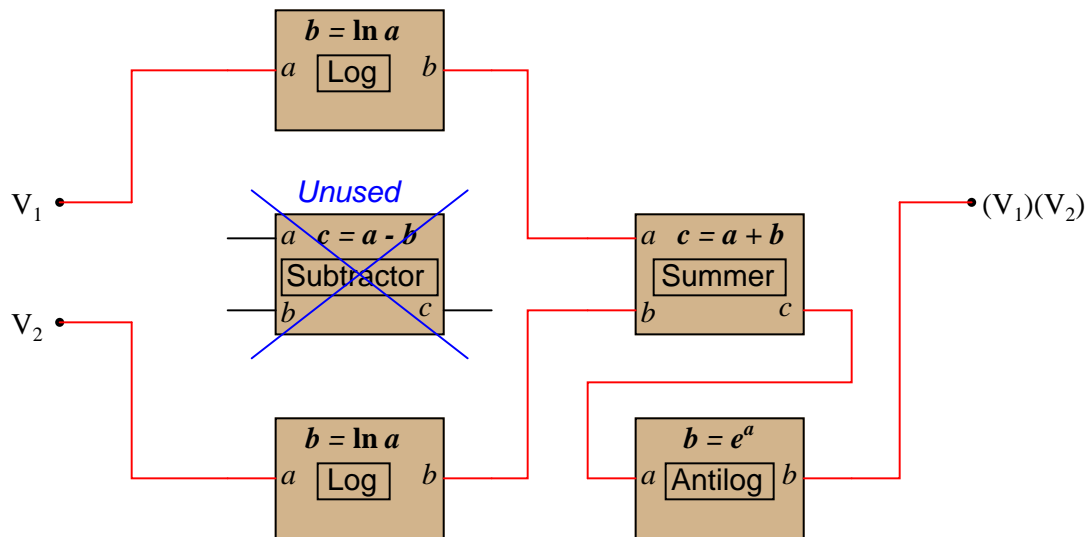
Notes 35

The purpose of this question is simple: to provide a practical application for logarithms as computational aids in an age of cheap, ubiquitous, digital computing devices.

Question 36

The relationship between voltage and current for a PN junction is described by this equation, sometimes referred to as the "diode equation," or "Shockley's diode equation" after its discoverer:

$$I_D = I_S(e^{\frac{qV_D}{NkT}} - 1)$$

Where,

I_D = Current through the PN junction, in amps

I_S = PN junction saturation current, in amps (typically 1 picoamp)

e = Euler's number ≈ 2.718281828

q = Electron unit charge, 1.6×10^{-19} coulombs

V_D = Voltage across the PN junction, in volts

N = Nonideality coefficient, or emission coefficient (typically between 1 and 2)

k = Boltzmann's constant, 1.38×10^{-23}

T = Junction temperature, degrees Kelvin

At first this equation may seem very daunting, until you realize that there are really only three variables in it: I_D, V_D, and T. All the other terms are constants. Since in most cases we assume temperature is fairly constant as well, we are really only dealing with two variables: diode current and diode voltage. Based on this realization, re-write the equation as a proportionality rather than an equality, showing how the two variables of diode current and voltage relate:

$$I_D \propto \; . \; . \; .$$

Based on this simplified equation, what would an I/V graph for a PN junction look like? How does this graph compare against the I/V graph for a resistor?
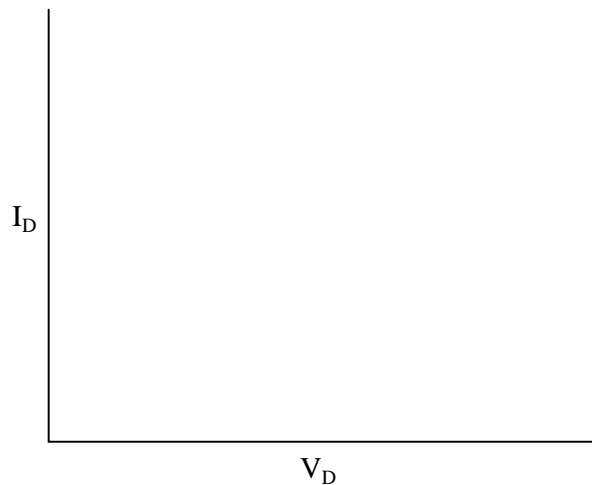
Answer 36

Simplified proportionality:

$$I_D \propto e^{V_D}$$

The graph described by the "diode formula" is a standard exponential curve, rising sharply as the independent variable (V_D, in this case) increases. The corresponding graph for a resistor, of course, is linear.

Notes 36

Ask your students to sketch their own renditions of an exponential curve on the whiteboard for all to see. Don't just let them get away with parroting the answer: "It's an exponential curve."

Question 37

Plot the transfer function (V_{out} versus V_{in}) for this opamp circuit, and explain how the circuit works:
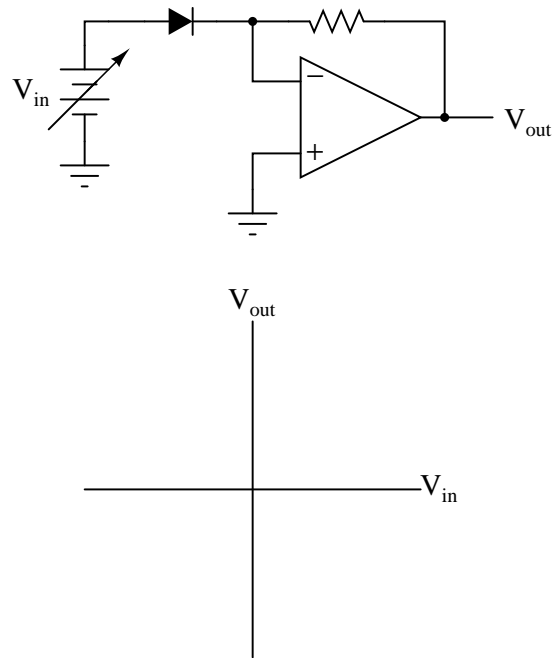
What type of mathematical function is represented by this circuit?
<u>file 01015</u>

Answer 37

This circuit represents an *exponential* function ($y \propto e^x$):
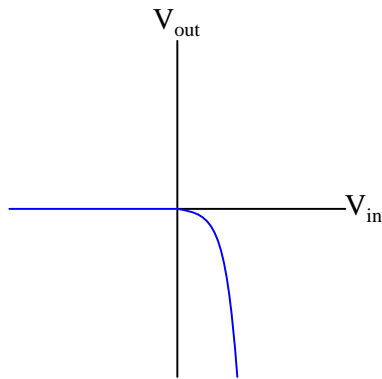
Notes 37

The direction of the transfer function curve may surprise some students. Ask them why the curve goes down (negative) for increasingly positive input voltages.

50

Question 38

We know that an opamp connected to a voltage divider with a voltage division ratio of $\frac{1}{2}$ will have an overall voltage gain of 2, and that the same circuit with a voltage division ratio of $\frac{2}{3}$ will have an overall voltage gain of 1.5, or $\frac{3}{2}$:
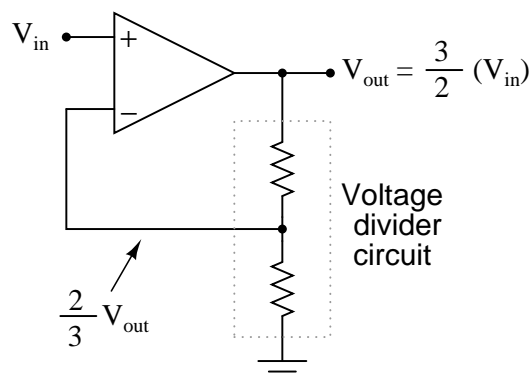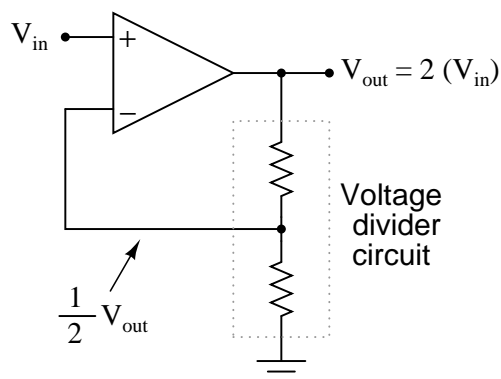
There is definitely a mathematical pattern at work in these noninverting opamp circuits: the overall voltage gain of the circuit is the *mathematical inverse* of the feedback network's voltage gain.

Building on this concept, what do you think would be the overall function of the following opamp circuits?

51

Answer 38
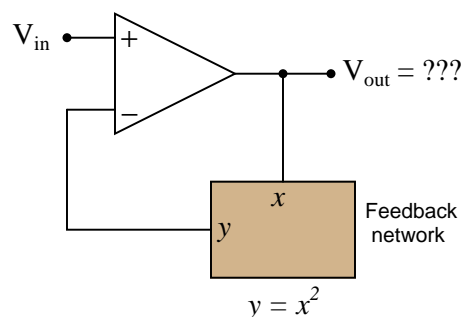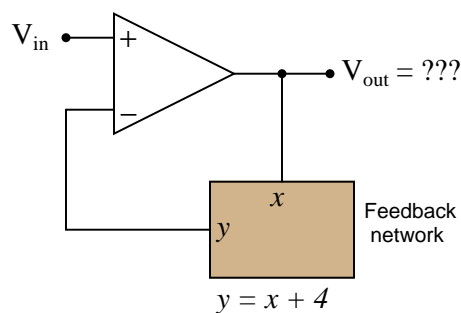
For the left-hand circuit: $V_{out} = V_{in} - 4$

For the right-hand circuit: $V_{out} = \sqrt{V_{in}}$

The result of placing a mathematical function in the feedback loop of a noninverting opamp circuit is that the output becomes the inverse function of the input: it literally becomes the value of x needed to solve for the input value of y:

$$y = f(x)$$

Notes 38
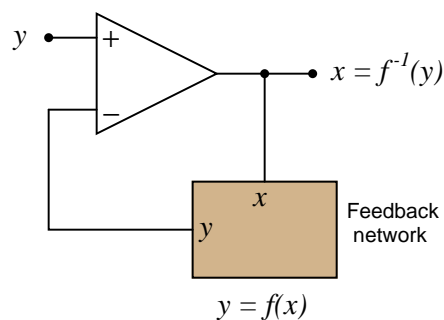
What is shown in this question and answer is a stark example of the power of negative feedback in a mathematical system. Here, we see the opamp's ability to solve for the input variable in an equation which we know the output value of. To state this in simpler terms, the opamp "does algebra" for us by "manipulating" the feedback network's equation to solve for x given an input signal of y.

Question 39

Plot the transfer function (V_{out} versus V_{in}) for this opamp circuit, and explain how the circuit works:
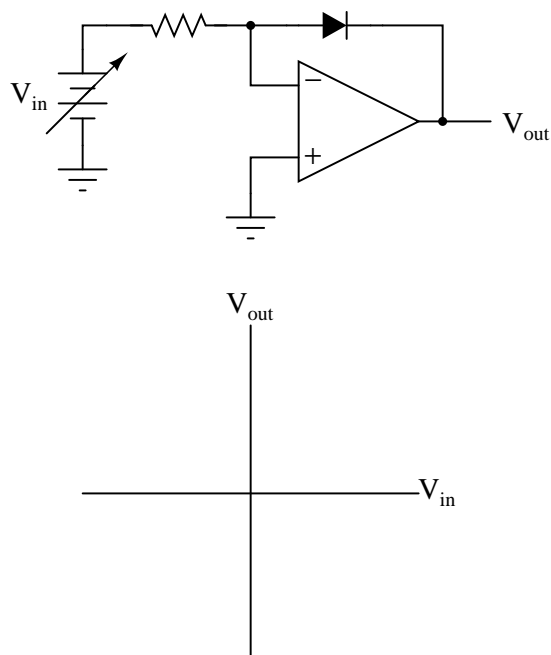
What type of mathematical function is represented by this circuit?

file 01016

Answer 39

This circuit represents a *logarithmic* function ($y \propto \ln x$):
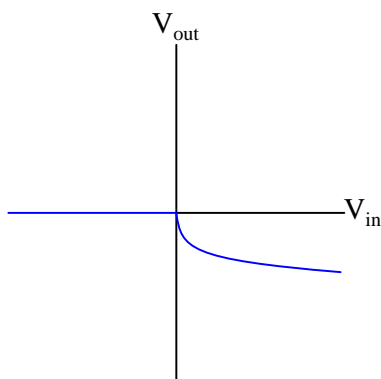
Notes 39

The direction of the transfer function curve may surprise some students. Ask them why the curve goes down (negative) for increasingly positive input voltages.

Ask your students how they obtained this transfer function curve. There are conceptual methods for obtaining it, as well as algebraic methods. It would be interesting to compare more than one of these methods in a class discussion, and have students gain insight from each others' methods.

Plot the transfer function (V_{out} versus V_{in}) for this opamp circuit:
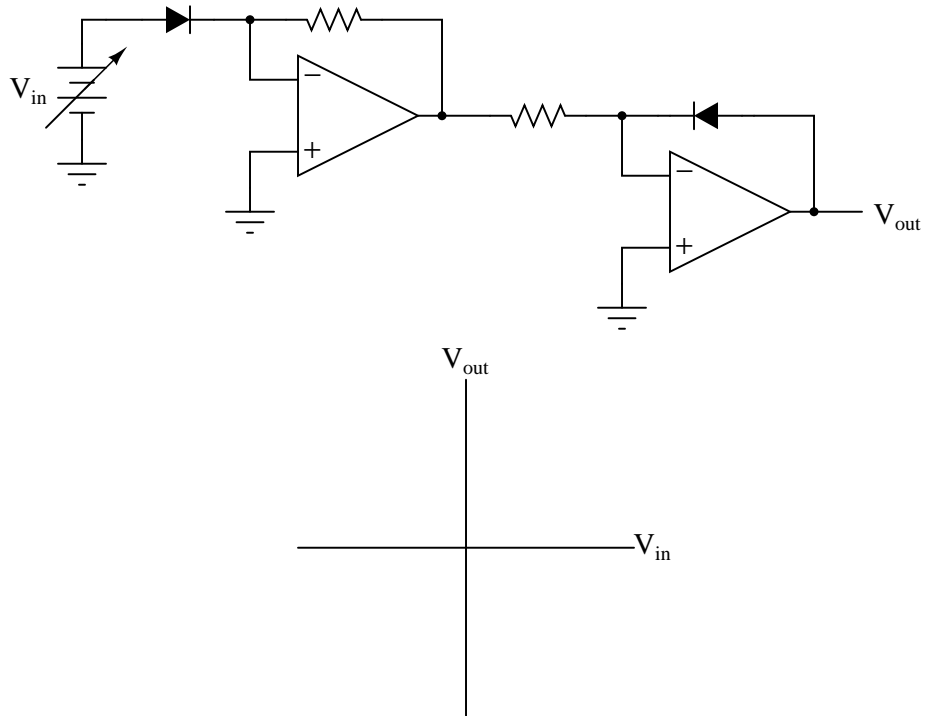
What type of mathematical function is represented by this circuit?
file 01017

Answer 40

This circuit (ideally) represents a *linear* function ($y \propto x$):
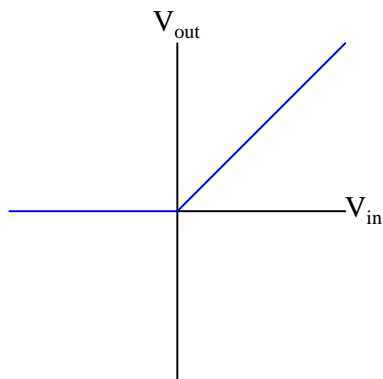
It should be obvious from inspection that the two opamp circuits represent inverse mathematical functions. Ask your students why the final transfer function is linear rather than nonlinear. After all, they should realize that each of the opamp circuits, taken individually, are very nonlinear. Why would their combined effect be linear?

An interesting exercise would be to have your students perform inverse functions like this on their hand calculators, first calculating an exponential function ($f(x) = e^x$), then a logarithmic ($g(x) = \ln x$), and verifying the combined functions' output ($f[g(x)] = x$).

Question 41

Identify the mathematical function of this circuit (if you look closely, you'll notice that the transistors are connected in such a way that they act very similar to diodes):
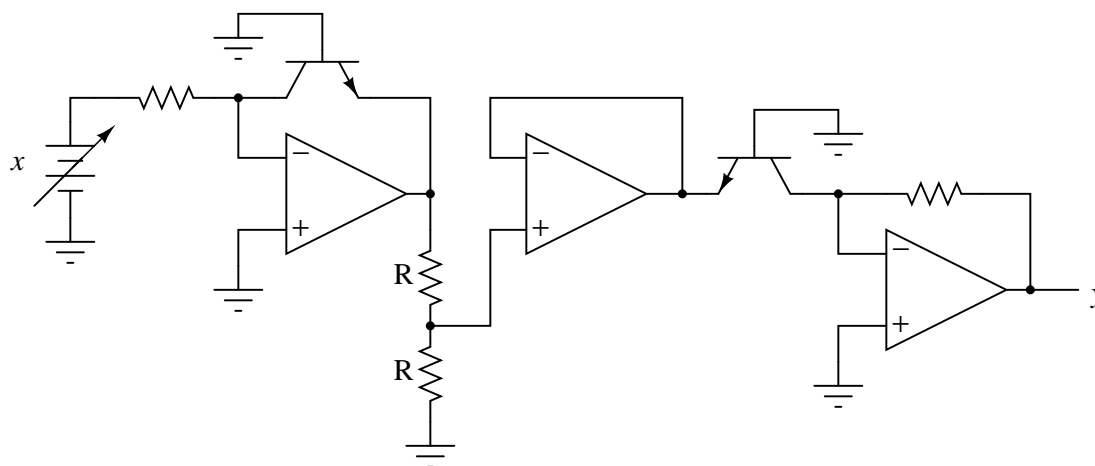
Note: the two resistors labeled "R" are equal in value.

file 02693

Answer 41

This circuit takes the *square root* of the input signal ($y = \sqrt{x}$).

Follow-up question: how could we modify this circuit so as to take the *cube root* of the input signal?

Notes 41

This circuit is not nearly as complex as it may appear at first, if students take the time to isolate it section-by-section and identify the mathematical function each section performs.

Question 42

Identify the mathematical function of this circuit (if you look closely, you'll notice that the transistors are connected in such a way that they act very similar to diodes):
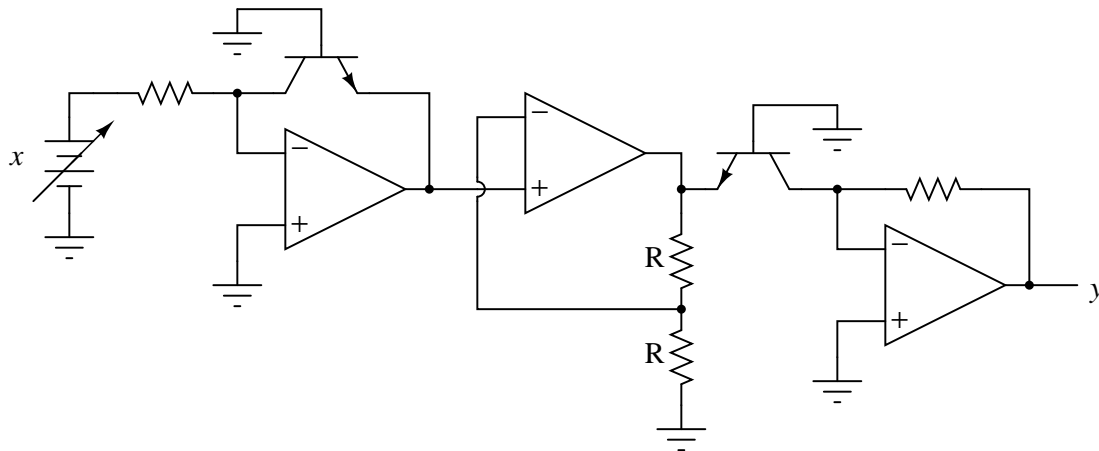
Note: the two resistors labeled "R" are equal in value.

file 01019

Answer 42

This circuit *squares* the input signal ($y = x^2$).

Challenge question: why are transistors used instead of diodes, since they have been effectively "disabled" to act as such?

Notes 42

This circuit is not nearly as complex as it may appear at first, if students take the time to isolate it section-by-section and identify the mathematical function each section performs.

Question 43

Suppose that in the course of building this exponential circuit you encounter severe inaccuracies: the circuit seems to work some of the time, but often its output deviates substantially (as much as +/- 10%) from what it ought to be:

Intended transfer function:

$$y = x^{0.5}$$
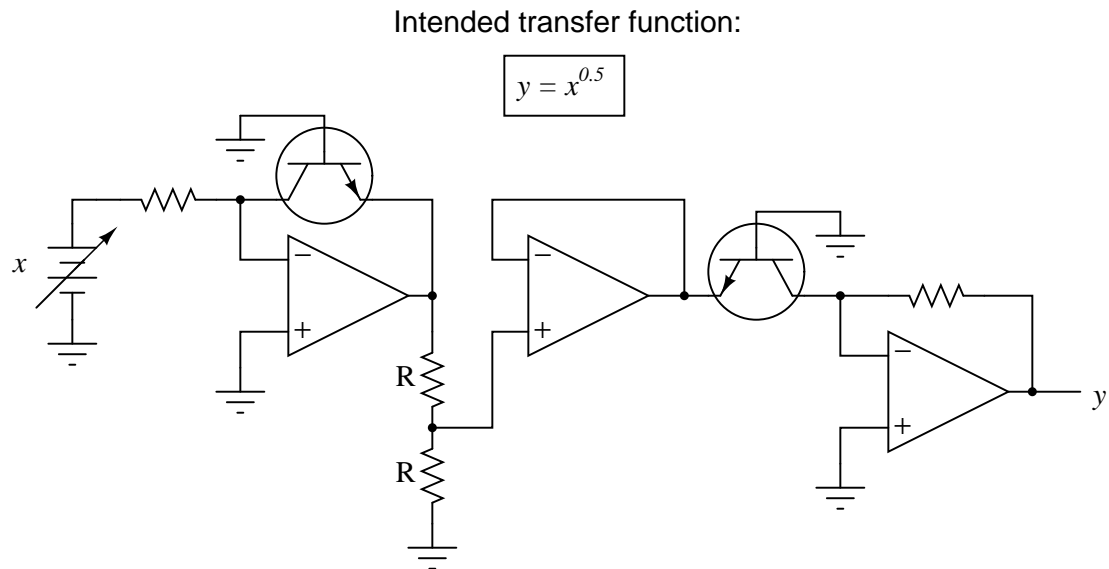
Based on what you know of the components in this circuit, what could be varying so much as to cause these errors? What do you recommend as a solution to the problem?

file 01020

Answer 43

The solution is to make sure both transistors are precisely matched, at held at the exact same temperature:
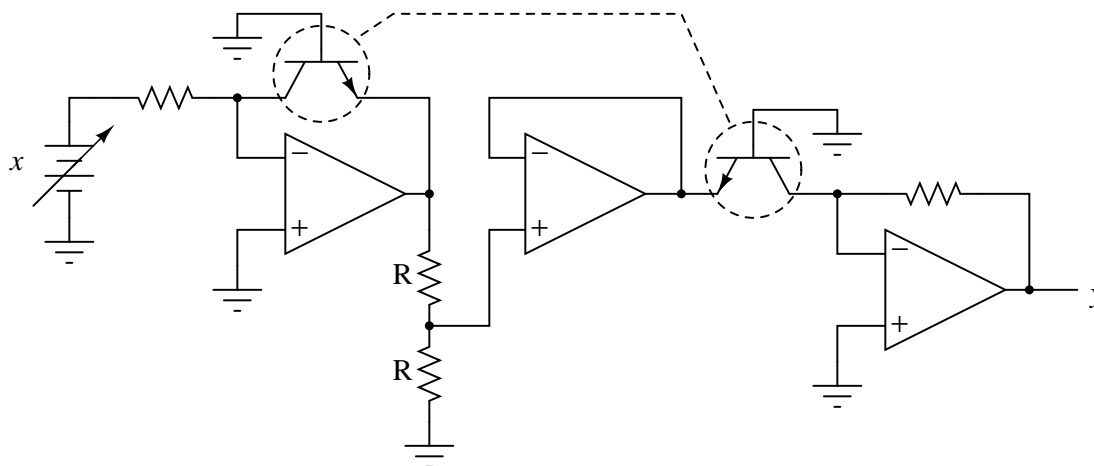
Challenge question: is there a part we could order that contains two matched, heat-stabilized transistors for an application such as this? Are there any other circuit applications you can think of that could benefit from using a precision-matched pair of transistors?

Notes 43

Ask your students to explain how they know temperature is an influencing factor in the accuracy of this circuit. Ask them to show any equations describing transistor behavior that demonstrate temperature dependence.

This question provides an opportunity to review the meaning of fractional exponents with your students. What, exactly, does $y = x^{0.5}$ mean? Ask your students to write this expression using more common symbols. Also, ask them what would have to be modified in this circuit to alter the exponent's value.

As for the challenge question, ask your students to produce a part number for the precision-matched transistor pair they find. Where did they obtain the information on this component?

Question 44

Design an op-amp circuit that *divides* one quantity (x) by another quantity (y) using logarithms. To give you a start on this circuit, I'll provide the initial logarithmic op-amp modules in this diagram:
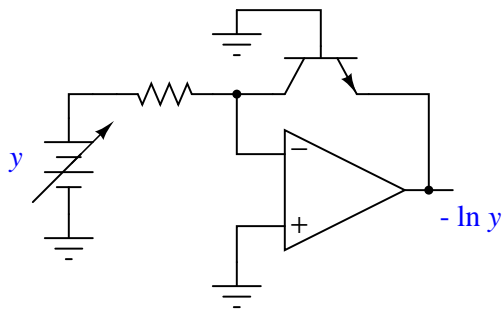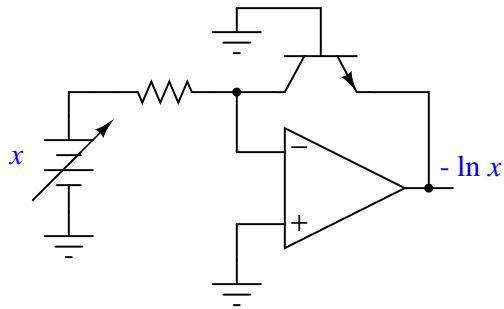
Note: it will be helpful for your analysis to write the mathematical expression at each op-amp output in your circuit, so you may readily see how the overall math function is constructed from individual steps.

file 01023
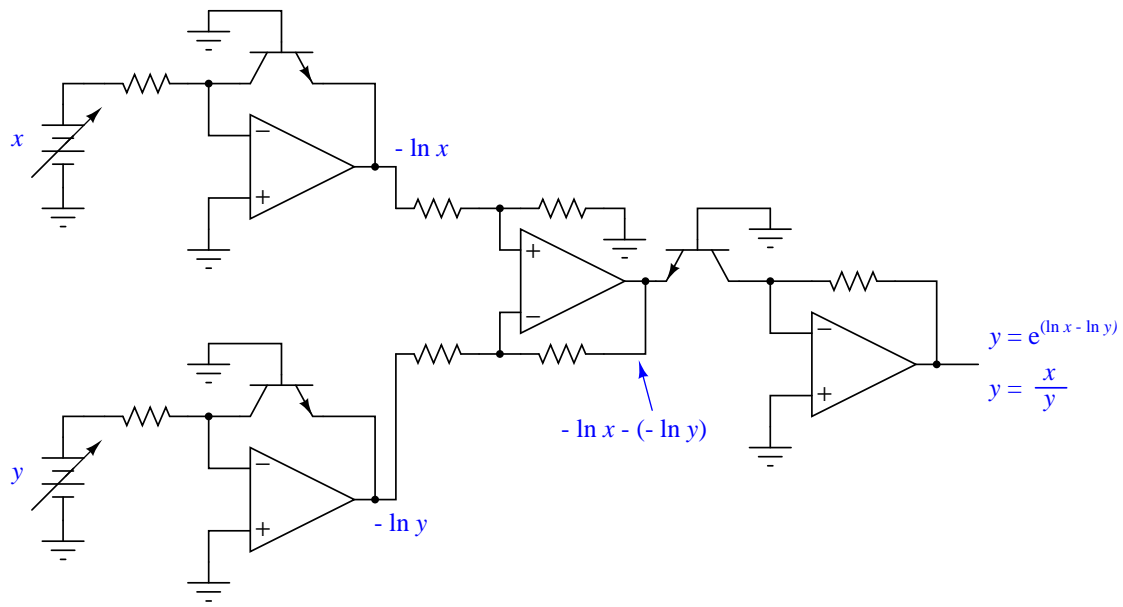
Answer 44

Notes 44

The circuit shown in the answer is a very common logarithmic construction: a *log-ratio* circuit, useful for many operations other than simple division. This question challenges students to put together the logarithm, antilogarithm, and differential op-amp circuits in a way that achieves the final design goal. Perhaps the most challenging aspect of this problem is managing the sign reversals.

Question 45

Find a datasheet for the AD538, an integrated circuit manufactured by Analog Devices. Then, read it carefully and explain how it is able to perform arithmetic functions such as multiplication, division, powers, and roots.

file 02692

Answer 45

I'll leave this up to you to research and to your classmates and instructor to discuss!

Notes 45

This question is destined for obsolescence, as one day the AD538 will no longer be manufactured. Until then, it is a fine piece of engineering, showcasing the power of logarithms as a computational aid in analog circuitry.

Competency: **Opamp relaxation oscillator**   Version:

Schematic

Given conditions

+V = R_1 = R_3 =

-V = R_2 = C_1 =

Parameters

	Predicted	Measured
V_{out} (pk-pk)		
f_{out}		

Fault analysis

Suppose component [   ] fails   ☐ open   ☐ other _____
                                    ☐ shorted

What will happen in the circuit?

file 02568

Answer 46

Use circuit simulation software to verify your predicted and measured parameter values.
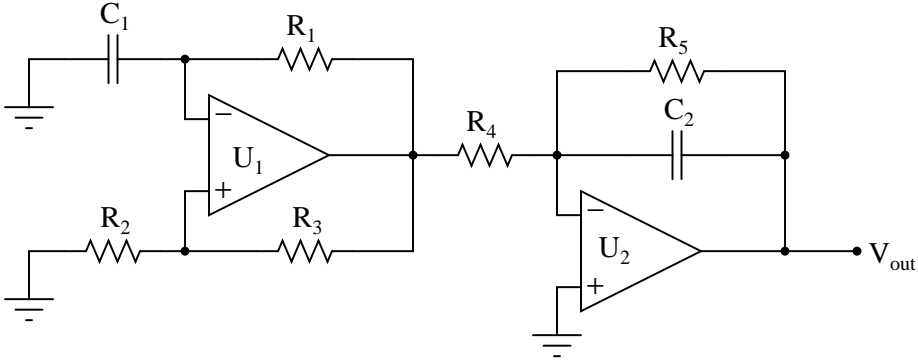
Notes 46

Use a dual-voltage, regulated power supply to supply power to the opamp. Specify standard resistor values, all between 1 kΩ and 100 kΩ (1k5, 2k2, 2k7, 3k3, 4k7, 5k1, 6k8, 10k, 22k, 33k, 39k 47k, 68k, etc.).

I have had good success using the following values:

- +V = +12 volts
- -V = -12 volts
- R_1 = 10 kΩ
- R_2 = 10 kΩ
- R_3 = 10 kΩ
- C_1 = 0.1 μF
- U_1 = one-half of LM1458 dual operational amplifier

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Competency: **Opamp triangle wave generator**   Version:

Schematic

Given conditions

$+V =$ $R_1 =$ $R_3 =$ $R_5 =$ $C_2 =$

$-V =$ $R_2 =$ $R_4 =$ $C_1 =$

Parameters

	Predicted	Measured
V_{out} (pk-pk)		
f_{out}		

Fault analysis

Suppose component ☐ fails   ☐ open   ☐ other _____

☐ shorted

What will happen in the circuit?

Answer 47

Use circuit simulation software to verify your predicted and measured parameter values.

Notes 47

Use a dual-voltage, regulated power supply to supply power to the opamp. Specify standard resistor values, all between 1 kΩ and 100 kΩ (1k5, 2k2, 2k7, 3k3, 4k7, 5k1, 6k8, 10k, 22k, 33k, 39k 47k, 68k, etc.).

I have had good success using the following values:

- +V = +12 volts
- -V = -12 volts
- R_1 = 10 kΩ
- R_2 = 10 kΩ
- R_3 = 10 kΩ
- R_4 = 10 kΩ
- R_5 = 100 kΩ
- C_1 = 0.1 μF
- C_2 = 0.47 μF
- U_1 = one-half of LM1458 dual operational amplifier
- U_2 = other half of LM1458 dual operational amplifier

It is a good idea to choose capacitor C_2 as a larger value than capacitor C_1, so that the second opamp does not saturate.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Competency: **Opamp LC resonant oscillator w/limiting**   Version:

Schematic

R_{pot}

D_1 R_2

U_1

V_{out}

C_1

D_2 R_3

R_1

L_1

Given conditions

+V = R_1 = L_1 = $R_2 = R_3$ =

-V = C_1 = R_{pot} =

Parameters

            Predicted   Measured

f_{out}

Calculations

file 02699

Answer 48

Use circuit simulation software to verify your predicted and measured parameter values.

Notes 48

Use a dual-voltage, regulated power supply to supply power to the opamp. Specify standard resistor values, all between 1 kΩ and 100 kΩ (1k5, 2k2, 2k7, 3k3, 4k7, 5k1, 6k8, 10k, 22k, 33k, 39k 47k, 68k, etc.).

I have had good success using the following values:

- $+V = +12$ volts
- $-V = -12$ volts
- $R_1 = 10$ kΩ
- $R_2 = R_3 = 1$ kΩ
- $R_{pot} = 10$ kΩ multi-turn
- $C_1 = 0.001$ μF or 0.47 μF
- $L_1 = 100$ mH
- $D_1 = D_2 = 1N4148$
- $U_1 =$ one-half of LM1458 dual operational amplifier

With the presence of the amplitude-limiting diodes D_1 and D_2, the potentiometer adjustment is not nearly as sensitive as without. Try removing both diodes to see what happens when there is no amplitude limiting at all! Students will have to finely adjust the multi-turn potentiometer to achieve a good sine wave (meeting the Barkhausen criterion). With the diodes in place, however, you may adjust the potentiometer for a loop gain just above unity with the only consequence being slight distortion of the waveform rather than severe distortion.

An extension of this exercise is to incorporate troubleshooting questions. Whether using this exercise as a performance assessment or simply as a concept-building lab, you might want to follow up your students' results by asking them to predict the consequences of certain circuit faults.

Question 49

NAME: _____   **Project Grading Criteria**   PROJECT: _____

You will receive the highest score for which *all* criteria are met.

100 % *(Must meet or exceed all criteria listed)*
  A. Impeccable craftsmanship, comparable to that of a professional assembly
  B. No spelling or grammatical errors anywhere in *any* document, upon first submission to instructor

95 % *(Must meet or exceed these criteria in addition to all criteria for 90% and below)*
  A. Technical explanation sufficiently detailed to teach from, inclusive of every component (supersedes 75.B)
  B. Itemized parts list complete with part numbers, manufacturers, and (equivalent) prices for *all* components, including recycled components and parts kit components (supersedes 90.A)

90 % *(Must meet or exceed these criteria in addition to all criteria for 85% and below)*
  A. Itemized parts list complete with prices of components purchased for the project, plus total price
  B. No spelling or grammatical errors anywhere in *any* document upon final submission

85 % *(Must meet or exceed these criteria in addition to all criteria for 80% and below)*
  A. "User's guide" to project function (in addition to 75.B)
  B. Troubleshooting log describing all obstacles overcome during development and construction

80 % *(Must meet or exceed these criteria in addition to all criteria for 75% and below)*
  A. All controls (switches, knobs, etc.) clearly and neatly labeled
  B. All documentation created on computer, not hand-written (including the schematic diagram)

75 % *(Must meet or exceed these criteria in addition to all criteria for 70% and below)*
  A. Stranded wire used wherever wires are subject to vibration or bending
  B. Basic technical explanation of all major circuit sections
  C. Deadline met for working prototype of circuit (Date/Time = _____ / _____)

70 % *(Must meet or exceed these criteria in addition to all criteria for 65%)*
  A. All wire connections sound (solder joints, wire-wrap, terminal strips, and lugs are all connected properly)
  B. No use of glue where a fastener would be more appropriate
  C. Deadline met for submission of fully-functional project (Date/Time = _____ / _____) – supersedes 75.C if final project submitted by that (earlier) deadline

65 % *(Must meet or exceed these criteria in addition to all criteria for 60%)*
  A. Project fully functional
  B. All components securely fastened so nothing is "loose" inside the enclosure
  C. Schematic diagram of circuit

60 % *(Must meet or exceed these criteria in addition to being safe and legal)*
  A. Project minimally functional, with all components located inside an enclosure (if applicable)
  B. Passes final safety inspection (proper case grounding, line power fusing, power cords strain-relieved)

0 % *(If <u>any</u> of the following conditions are true)*
  A. Fails final safety inspection (improper grounding, fusing, and/or power cord strain relieving)
  B. Intended project function poses a safety hazard
  C. Project function violates any law, ordinance, or school policy
   file 03173

Answer 49

Be sure you meet with your instructor if you have any questions about what is expected for your project!

Notes 49

The purpose of this assessment rubric is to act as a sort of "contract" between you (the instructor) and your student. This way, the expectations are all clearly known in advance, which goes a long way toward disarming problems later when it is time to grade.

Question 50

Predict how the operation of this relaxation oscillator circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no multiple faults):
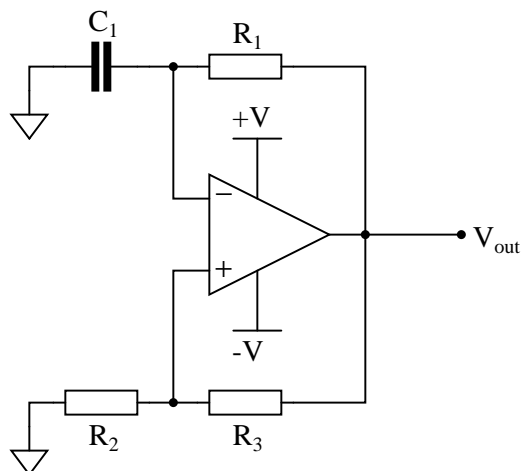
- Resistor R_1 fails open:

- Solder bridge (short) across resistor R_1:

- Capacitor C_1 fails shorted:

- Solder bridge (short) across resistor R_2:

- Resistor R_3 fails open:

  For each of these conditions, explain *why* the resulting effects will occur.

  file 03798

Answer 50

- Resistor R_1 fails open: *Opamp output saturates either positive or negative.*

- Solder bridge (short) across resistor R_1: *Output voltage settles to 0 volts.*

- Capacitor C_1 fails shorted: *Opamp output saturates either positive or negative.*

- Solder bridge (short) across resistor R_2: *Output voltage settles to 0 volts.*

- Resistor R_3 fails open: *Output voltage settles to 0 volts.*

Notes 50

The purpose of this question is to approach the domain of circuit troubleshooting from a perspective of knowing what the fault is, rather than only knowing what the symptoms are. Although this is not necessarily a realistic perspective, it helps students build the foundational knowledge necessary to diagnose a faulted circuit from empirical data. Questions such as this should be followed (eventually) by other questions asking students to identify likely faults based on measurements.

Question 51

Identify at least two different component faults that would result in a change in duty cycle for this oscillator circuit:
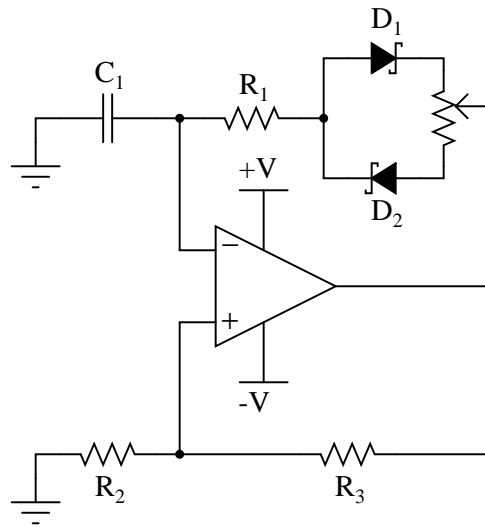
file 03799

Answer 51

A short in either of the two diodes would cause the duty cycle to change.

Follow-up question: what would happen if either of these two diodes failed open?

Notes 51

Ask your students to explain why the duty cycle would change as a result of either diode failing shorted. This is a good opportunity to further explore the operation of this oscillator circuit.

Question 52

Suppose this LC oscillator stopped working, and you suspected either the capacitor or the inductor as having failed. How could you check these two components *without* the use of an LCR meter?
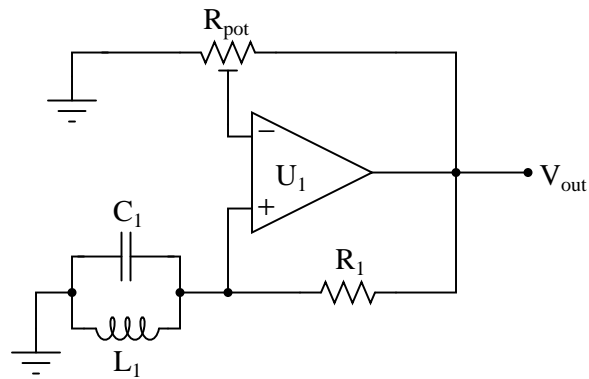
file 03800

Answer 52

Try using an ohmmeter on each component, removed from the circuit. The capacitor should read "open" and the inductor should read a low resistance, if both are in good condition.

Follow-up question: is an ohmmeter test comprehensive enough to detect all possible faults with these two types of components? Why or why not? Be as specific as you can in your answer(s).

Notes 52

Knowing how to check the condition of components with primitive test equipment is a valuable skill. It is well worth your time to discuss this question (and its answers) with your students in detail, so they all understand the concepts involved.

Question 53

The purpose of this circuit is to provide a pushbutton-adjustable voltage. Pressing one button causes the output voltage to increase, while pressing the other button causes the output voltage to decrease. When neither button is pressed, the voltage remains stable. Essentially, it is an active *integrator* circuit, with resistors R_1 and R_2 and capacitor C_1 setting the rates of integration:
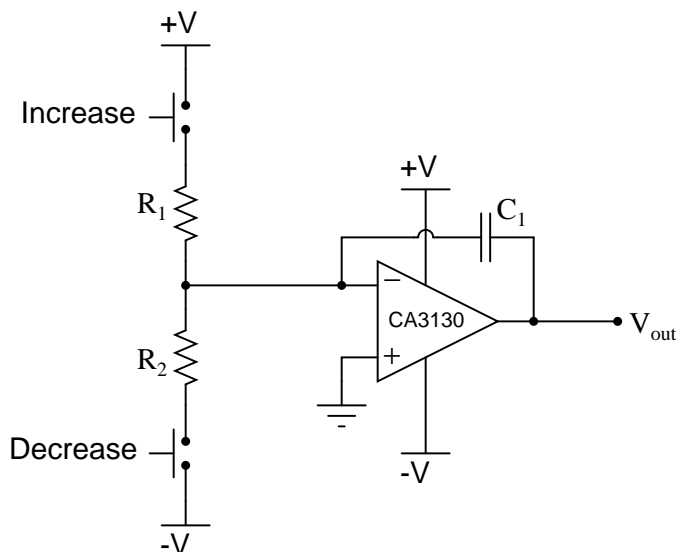
After working just fine for quite a long while, the circuit suddenly fails: now it only outputs zero volts DC all the time. Identify at least one component or wiring failure that would account for the circuit's faulty behavior, and explain why that fault would cause the circuit to act like this.

file 03802

Answer 53

The wire connecting the junction of R_1 and R_2 with the inverting input of the opamp could be broken open. Also, C_1 could be failed shorted.

Challenge question: why do you suppose I specify a CA3130 operational amplifier for this particular circuit? What is special about this opamp that qualifies it for the task?

Notes 53

I purposely omitted the explanations for the possible faults listed in the answer, because I want the students to think this through, and to do so in class with you present so you may hear their reasoning.

Question 54

This active integrator circuit processes the voltage signal from an *accelerometer*, a device that outputs a DC voltage proportional to its physical acceleration. The accelerometer is being used to measure the acceleration of an athlete's foot as he kicks a ball, and the job of the integrator is to convert that acceleration signal into a *velocity* signal so the researchers can record the velocity of the athlete's foot:

Active integrator circuit
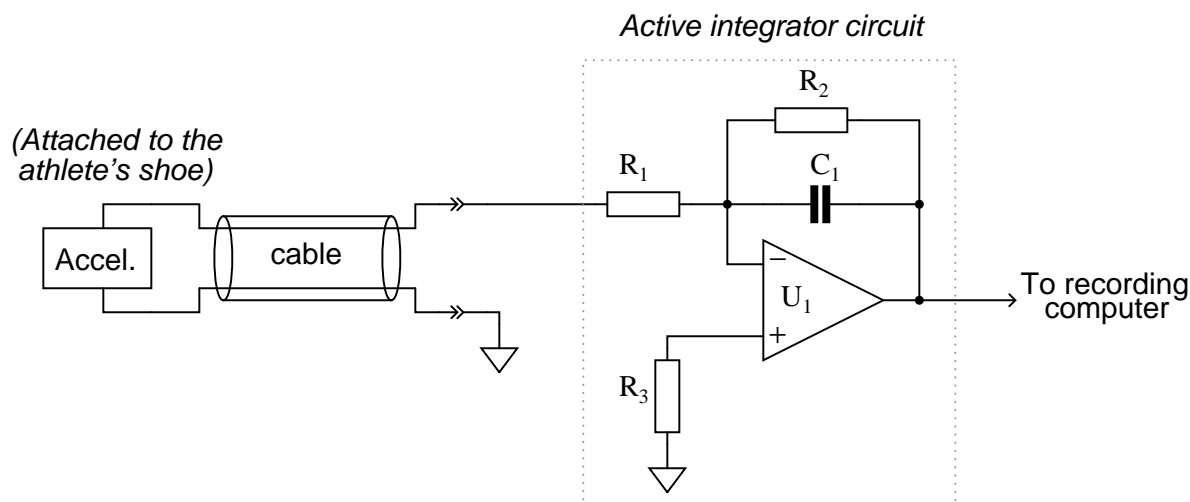
During the set-up for this test, a radar gun is used to check the velocity of the athlete's foot as he does come practice kicks, and compare against the output of the integrator circuit. What the researchers find is that the integrator's output is reading a bit low. In other words, the integrator circuit is not integrating fast enough to provide an accurate representation of foot velocity.

Determine which component(s) in the integrator circuit may have been improperly sized to cause this calibration problem. Be as specific as you can in your answer(s).

file 03808

Answer 54

Resistor R_1 may be too large, and/or capacitor C_1 may be too large.

Notes 54

This is an interesting, practical question regarding the use of an integrator circuit for real-life signal processing. Ask your students to explain their reasoning as they state their proposed component faults.

Incidentally, if anyone asks what the purpose of R_2 or R_3 is, tell them that both are used for *opamp bias current compensation*. An ideal opamp would not require these components to be in place.

Question 55

Predict how the operation of this exponentiator circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no multiple faults):
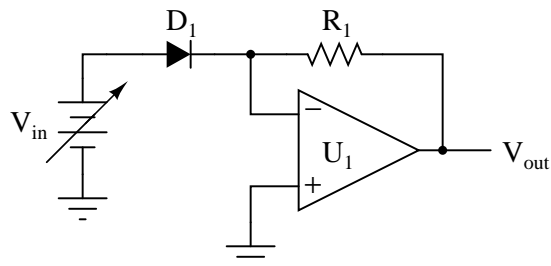
- Resistor R_1 fails open:
- Solder bridge (short) across resistor R_1:
- Diode D_1 fails open:
- Diode D_1 fails shorted:

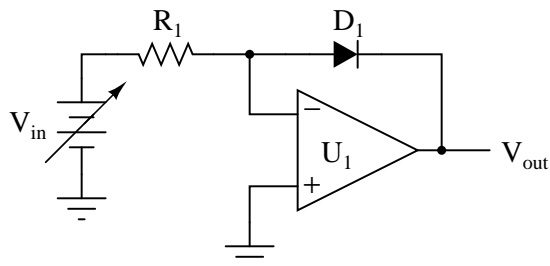For each of these conditions, explain *why* the resulting effects will occur.

file 03803

Answer 55

- Resistor R_1 fails open: *V_{out} saturates in the negative direction.*
- Solder bridge (short) across resistor R_1: *V_{out} goes to zero volts.*
- Diode D_1 fails open: *V_{out} goes to zero volts.*
- Diode D_1 fails shorted: *V_{out} saturates in the negative direction.*

Notes 55

The purpose of this question is to approach the domain of circuit troubleshooting from a perspective of knowing what the fault is, rather than only knowing what the symptoms are. Although this is not necessarily a realistic perspective, it helps students build the foundational knowledge necessary to diagnose a faulted circuit from empirical data. Questions such as this should be followed (eventually) by other questions asking students to identify likely faults based on measurements.

Question 56

Predict how the operation of this logarithm extractor circuit will be affected as a result of the following faults. Consider each fault independently (i.e. one at a time, no multiple faults):

- Resistor R_1 fails open:
- Solder bridge (short) across resistor R_1:
- Diode D_1 fails open:
- Diode D_1 fails shorted:

    For each of these conditions, explain *why* the resulting effects will occur.
    file 03804

Answer 56

- Resistor R_1 fails open: V_{out} *goes to zero volts.*
- Solder bridge (short) across resistor R_1: V_{out} *saturates in the negative direction.*
- Diode D_1 fails open: V_{out} *saturates in the negative direction.*
- Diode D_1 fails shorted: V_{out} *goes to zero volts.*

Notes 56

The purpose of this question is to approach the domain of circuit troubleshooting from a perspective of knowing what the fault is, rather than only knowing what the symptoms are. Although this is not necessarily a realistic perspective, it helps students build the foundational knowledge necessary to diagnose a faulted circuit from empirical data. Questions such as this should be followed (eventually) by other questions asking students to identify likely faults based on measurements.

Question 57

Explain why carefully matched resistors and transistors are necessary in log/antilog circuits, using your own words. Also, explain why the operational amplifiers themselves need not be matched as precisely as the discrete components.

file 03807

Answer 57

I said, "your own words," not mine! What are you looking here for?

Notes 57

This question challenges students to identify the "weak points" of log/antilog circuits, explaining *why* certain component tolerances are critical and others are not. This is a good test of students' understanding regarding log/antilog circuits and their underlying theory.

Question 58

This square root extraction circuit used to work fine, but then one day it stopped outputting the square root of the input signal, and instead simply reproduced the input signal with a gain of 1:
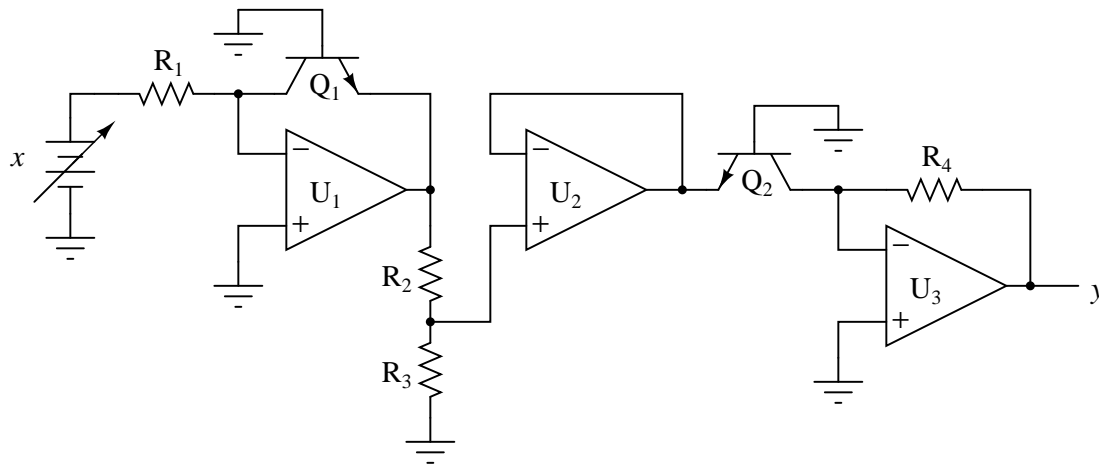
What could have gone wrong with this circuit to cause it to stop "calculating" the square root of the input? Be as detailed as possible in your answer.

file 03805

Answer 58

Possibly R_2 failed shorted or R_3 failed open. I'll let you explain why either of these faults could cause the problem described.

Notes 58

One must understand what is happening in this circuit and why in order to successfully diagnose the problem. Discuss this carefully with your students.

Question 59

Identify at least two independent component faults which could cause this square root extraction circuit to always output 0 volts instead of the square root of input voltage x as it should:
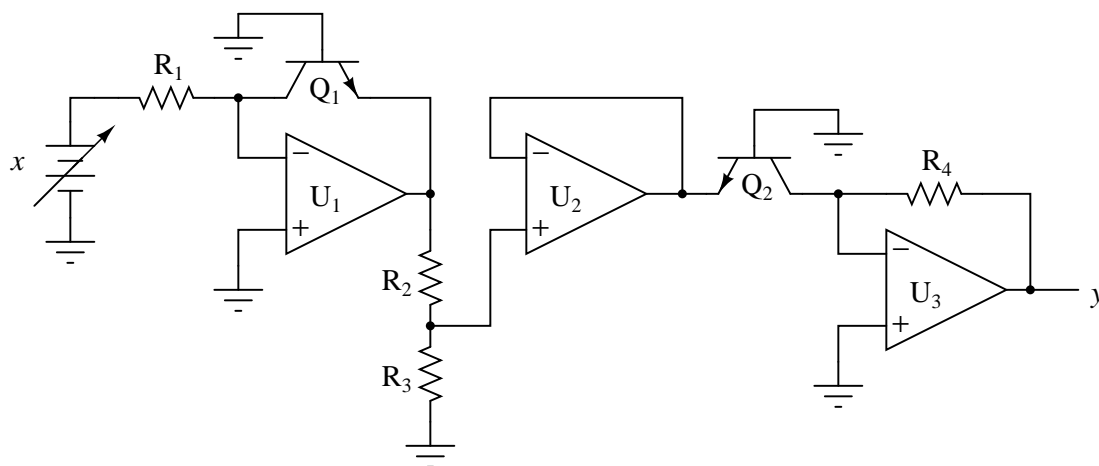
Explain why each of your proposed faults would cause the output to stay at 0 volts.

file 03806

Answer 59

Here are just a few possibilities: resistor R_2 failed open, solder bridge across resistor R_3, Q_2 failed open.

Notes 59

Students will likely identify opamp U_3 as a potential fault, but this is really too easy. Encourage them to seek more *interesting* fault possibilities!

Question 60

Most operational amplifiers do not have the ability to swing their output voltages rail-to-rail. Most of those do not swing their output voltages symmetrically. That is, a typical non-rail-to-rail opamp may be able to approach one power supply rail voltage closer than the other; e.g. when powered by a +15/-15 volt split supply, the output saturates positive at +14 volts and saturates negative at -13.5 volts.

What effect do you suppose this non-symmetrical output range will have on a typical relaxation oscillator circuit such as the following, and how might you suggest we fix the problem?
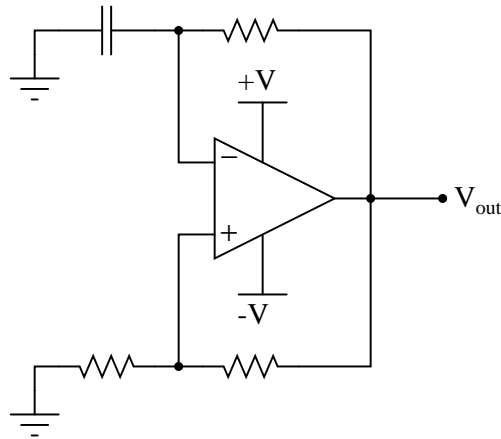
file 02675

Answer 60

The duty cycle will not be 50%. One way to fix the problem is to do something like this:
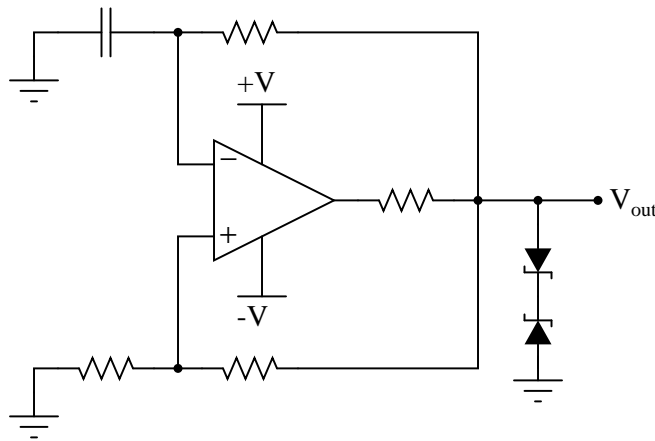
Follow-up question: explain how and why *this* solution works. Now you just *knew* I was going to ask this question the moment you saw the diagram, didn't you?

Notes 60

Note that I added an additional resistor to the circuit, in series with the opamp output terminal. In some cases this is not necessary because the opamp is self-limiting in output current, but it is a good design practice nonetheless. In the event anyone ever swaps out the original opamp for a different model lacking overcurrent protection, the new opamp will not become damaged.

81

Question 61

This resonant LC oscillator circuit is very sensitive to changes in the gain. Note how the potentiometer used in this circuit is the "trimmer" variety, adjustable with a screwdriver rather than by a knob or other hand control:

The reason for this choice in potentiometers is to make accidental changes in circuit gain less probable. If you build this circuit, you will see that tiny changes in this potentiometer's setting make a huge difference in the quality of the output sine wave. A little too much gain, and the sine wave becomes noticeably distorted. Too little gain, and the circuit stops oscillating altogether!

Obviously, it is not good to have such sensitivity to minor changes in any practical circuit expected to reliably perform day after day. One solution to this problem is to add a *limiting network* to the circuit comprised of two diodes and two resistors:

With this network in place, the circuit gain may be adjusted well above the threshold for oscillation (Barkhausen criterion) without exhibiting excessive distortion as it would have without the limiting network. Explain why the limiting network makes this possible.

file 03760

Answer 61

The limiting network attenuates the circuit gain as peak voltage begins to exceed 0.7 volts. This attenuation helps to prevent the opamp from clipping.

Follow-up question: what effect does this "limiting network" have on the purity of the oscillator's output signal spectrum? In other words, does the limiting network increase or decrease the harmonic content of the output waveform?

82

This circuit is important for students to encounter, as it reveals a very practical limitation of the "textbook" version of the resonant oscillator circuit. It is not enough that a circuit design work in ideal conditions – a practical circuit must be able to tolerate some variance in component values or else it will not operate reliably.

Question 62

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Calculus is a branch of mathematics that originated with scientific questions concerning *rates of change*. The easiest rates of change for most people to understand are those dealing with time. For example, a student watching their savings account dwindle over time as they pay for tuition and other expenses is very concerned with rates of change (*dollars per year* being spent).

In calculus, we have a special word to describe rates of change: *derivative*. One of the notations used to express a derivative (rate of change) appears as a fraction. For example, if the variable S represents the amount of money in the student's savings account and t represents time, the rate of change of dollars over time would be written like this:

$$\frac{dS}{dt}$$

The following set of figures puts actual numbers to this hypothetical scenario:

- Date: November 20

- Saving account balance (S) = \$12,527.33

- Rate of spending $\left(\frac{dS}{dt}\right)$ = -5,749.01 per year

List some of the equations you have seen in your study of electronics containing derivatives, and explain how *rate of change* relates to the real-life phenomena described by those equations.

<u>file 03310</u>

Answer 62

Voltage and current for a capacitor:

$$i = C\frac{dv}{dt}$$

Voltage and current for an inductor:

$$v = L\frac{di}{dt}$$

Electromagnetic induction:

$$v = N\frac{d\phi}{dt}$$

I leave it to you to describe how the rate-of-change over time of one variable relates to the other variables in each of the scenarios described by these equations.

Follow-up question: why is the derivative quantity in the student's savings account example expressed as a negative number? What would a positive $\frac{dS}{dt}$ represent in real life?

Challenge question: describe actual circuits you could build to demonstrate each of these equations, so that others could see what it means for one variable's rate-of-change over time to affect another variable.

The purpose of this question is to introduce the concept of the derivative to students in ways that are familiar to them. Hopefully the opening scenario of a dwindling savings account is something they can relate to!

A very important aspect of this question is the discussion it will engender between you and your students regarding the relationship between rates of change in the three equations given in the answer. It is very important to your students' comprehension of this concept to be able to verbally describe how the derivative works in each of these formulae. You may want to have them phrase their responses in realistic terms, as if they were describing how to set up an illustrative experiment for a classroom demonstration.

Question 63

$\int f(x)\, dx$ *Calculus alert!*

Define what "derivative" means when applied to the graph of a function. For instance, examine this graph:
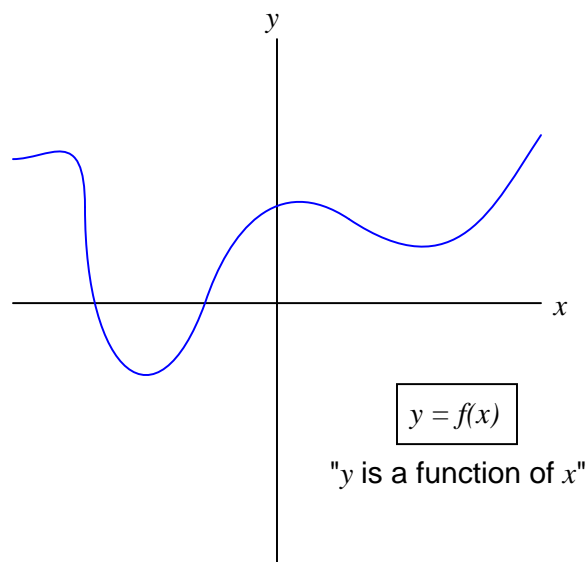
Label all the points where the derivative of the function $\left(\frac{dy}{dx}\right)$ is positive, where it is negative, and where it is equal to zero.

file 03646

Answer 63

The graphical interpretation of "derivative" means the *slope* of the function at any given point.
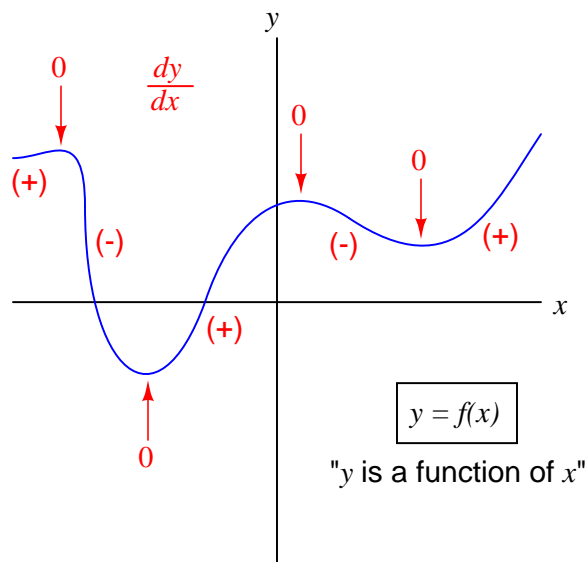
Usually students find the concept of the derivative easiest to understand in graphical form: being the *slope* of the graph. This is true whether or not the independent variable is time (an important point given that most "intuitive" examples of the derivative are time-based!).

Question 64

Shown here is the graph for the function $y = x^2$:

y

$$y = x^2$$

x
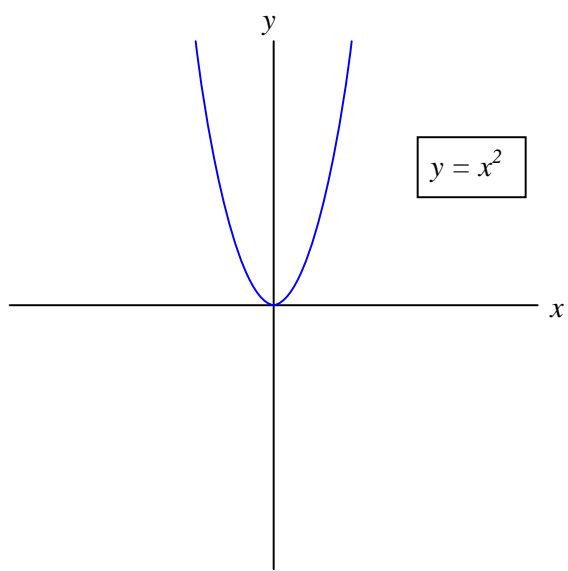
Sketch an approximate plot for the *derivative* of this function.

file 03647

Answer 64
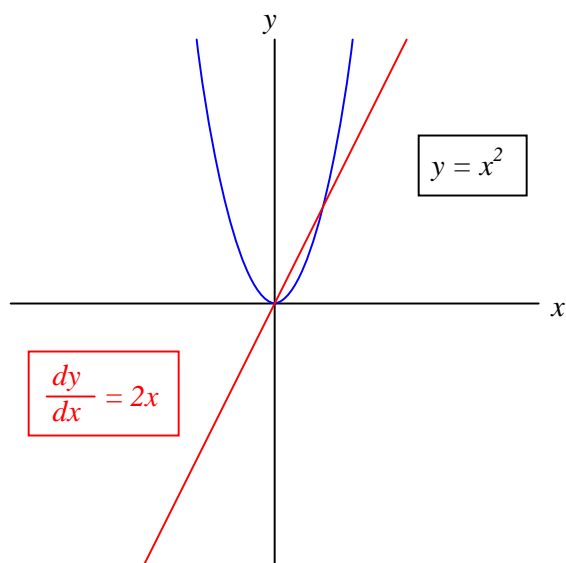
Challenge question: derivatives of power functions are easy to determine if you know the procedure. In this case, the derivative of the function $y = x^2$ is $\frac{dy}{dx} = 2x$. Examine the following functions and their derivatives to see if you can recognize the "rule" we follow:

- $y = x^3$ $\frac{dy}{dx} = 3x^2$

- $y = x^4$ $\frac{dy}{dx} = 4x^3$

- $y = 2x^4$ $\frac{dy}{dx} = 8x^3$

- $y = 10x^5$ $\frac{dy}{dx} = 50x^4$

- $y = 2x^3 + 5x^2 - 7x$ $\frac{dy}{dx} = 6x^2 + 10x - 7$

- $y = 5x^3 - 2x - 16$ $\frac{dy}{dx} = 15x^2 - 2$

- $y = 4x^7 - 6x^3 + 9x + 1$ $\frac{dy}{dx} = 28x^6 - 18x^2 + 9$

Notes 64

Usually students find the concept of the derivative easiest to understand in graphical form: being the *slope* of the graph. This is true whether or not the independent variable is time (an important point given that most "intuitive" examples of the derivative are time-based!).

Even if your students are not yet familiar with the power rule for calculating derivatives, they should be able to tell that $\frac{dy}{dx}$ is zero when $x = 0$, positive when $x > 0$, and negative when $x < 0$.

Question 65

$\boxed{\int f(x)\,dx \quad \text{Calculus alert!}}$

According to the "Ohm's Law" formula for a capacitor, capacitor current is proportional to the *time-derivative* of capacitor voltage:

$$i = C\frac{dv}{dt}$$

Another way of saying this is to state that the capacitors *differentiate* voltage with respect to time, and express this *time-derivative* of voltage as a current.

Suppose we had an oscilloscope capable of directly measuring current, or at least a current-to-voltage converter that we could attach to one of the probe inputs to allow direct measurement of current on one channel. With such an instrument set-up, we could directly plot capacitor voltage and capacitor current together on the same display:
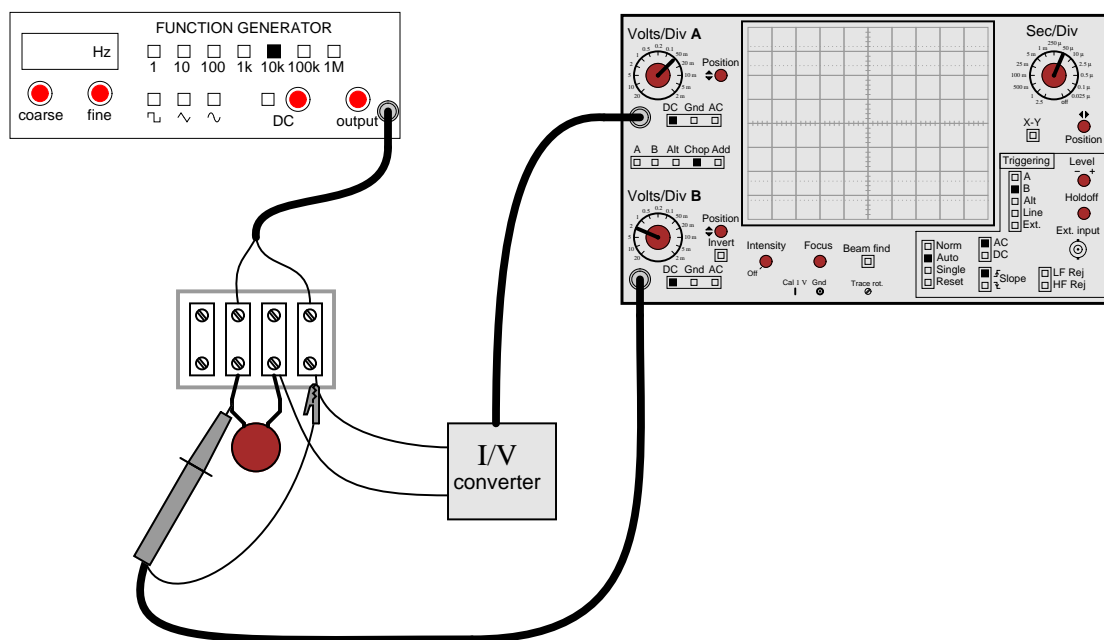
For each of the following voltage waveforms (channel B), plot the corresponding capacitor current waveform (channel A) as it would appear on the oscilloscope screen:
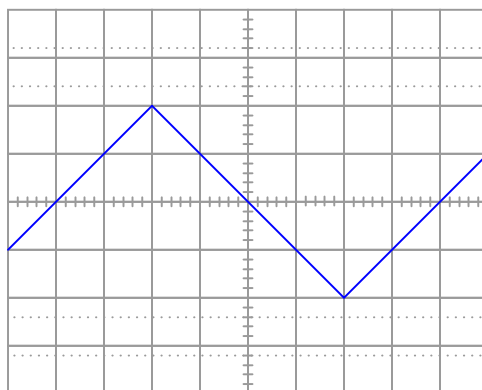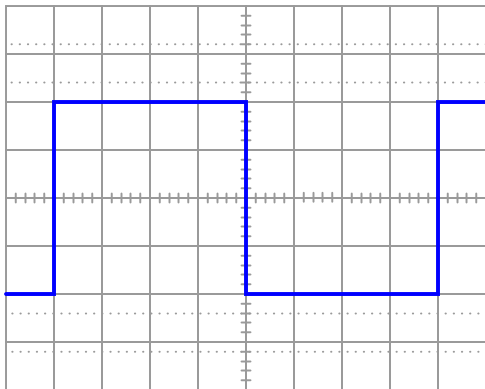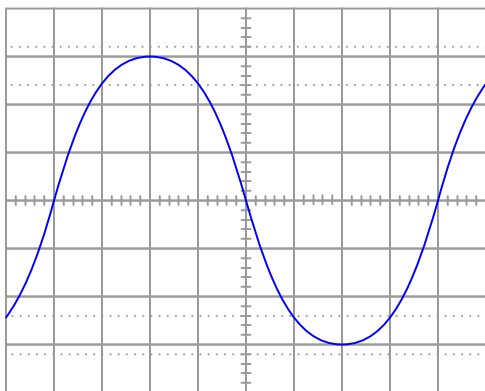
Note: the amplitude of your current plots is arbitrary. What I'm interested in here is the *shape* of each current waveform!

file 01900
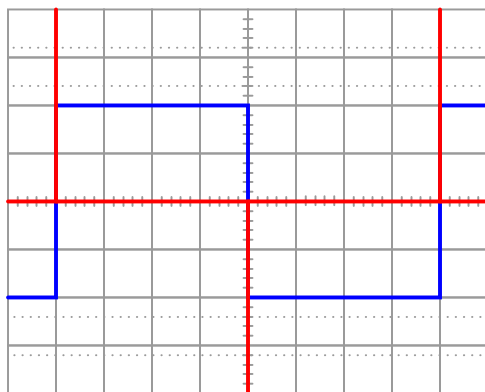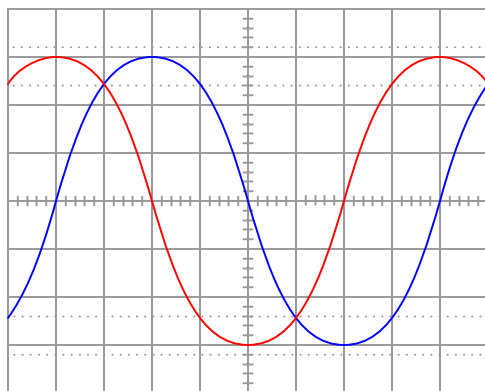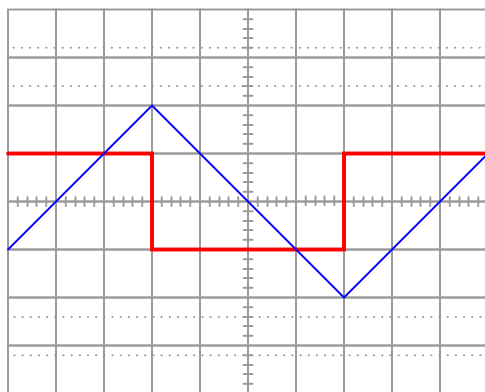
Follow-up question: what electronic device could perform the function of a "current-to-voltage converter" so we could use an oscilloscope to measure capacitor current? Be as specific as you can in your answer.

Notes 65

Here, I ask students to relate the instantaneous rate-of-change of the voltage waveform to the instantaneous amplitude of the current waveform. Just a conceptual exercise in derivatives.

Question 66

$\boxed{\int f(x)\, dx \quad \textit{Calculus alert!}}$

According to the "Ohm's Law" formula for a capacitor, capacitor current is proportional to the *time-derivative* of capacitor voltage:

$$i = C\frac{dv}{dt}$$

Another way of saying this is to state that the capacitors *differentiate* voltage with respect to time, and express this *time-derivative* of voltage as a current.

We may build a simple circuit to produce an output voltage proportional to the current through a capacitor, like this:

Passive differentiator circuit

C

R_{shunt} V_{out}
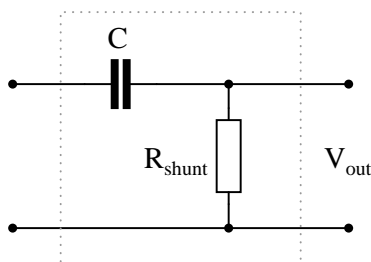
The resistor is called a *shunt* because it is designed to produce a voltage proportional to current, for the purpose of a parallel ("shunt")-connected voltmeter or oscilloscope to measure that current. Ideally, the shunt resistor is there only to help us measure current, and not to impede current through the capacitor. In other words, its value in ohms should be very small compared to the reactance of the capacitor ($R_{shunt} << X_C$).

Suppose that we connect AC voltage sources with the following wave-shapes to the input of this passive differentiator circuit. Sketch the ideal (time-derivative) output waveform shape on each oscilloscope screen, as well as the shape of the actual circuit's output voltage (which will be non-ideal, of course):
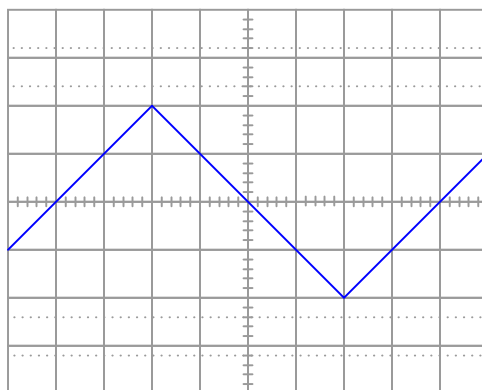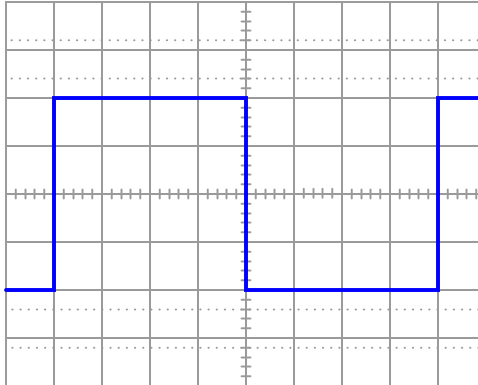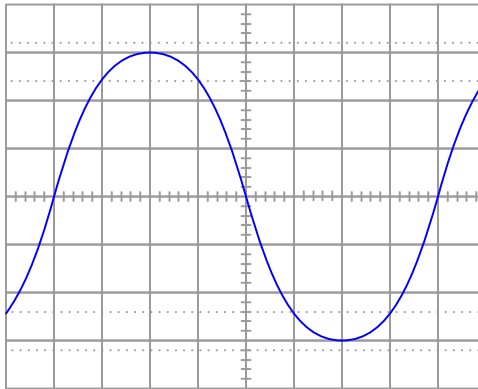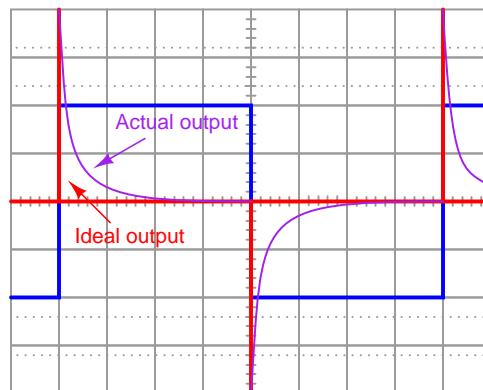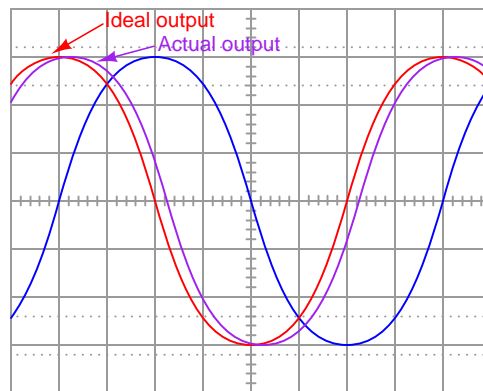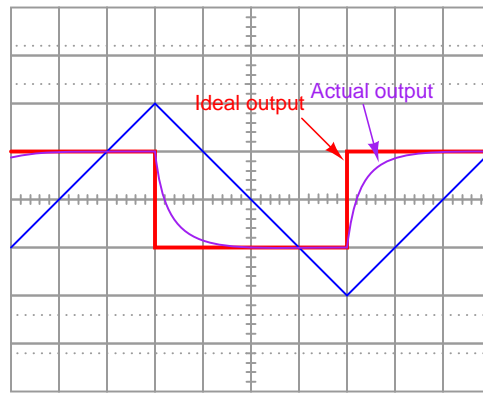
Note: the amplitude of your plots is arbitrary. What I'm interested in here is the *shape* of the ideal and actual output voltage waveforms!

Hint: I strongly recommend building this circuit and testing it with triangle, sine, and square-wave input voltage signals to obtain the corresponding actual output voltage wave-shapes!

<u>file 03643</u>

94

Follow-up question: given that $R_{shunt} << X_C$ in order that the resistance does not impede the capacitor current to any significant extent, what does this suggest about the necessary time-constant (τ) of a passive differentiator circuit? In other words, what values of R and C would work best in such a circuit to produce an output waveform that is as close to ideal as possible?

This question really is best answered by experimentation. I recommend having a signal generator and oscilloscope on-hand in the classroom to demonstrate the operation of this passive differentiator circuit. Challenge students with setting up the equipment and operating it!

Question 67

$\int f(x)\,dx$ *Calculus alert!*

Calculus is widely (and falsely!) believed to be too complicated for the average person to understand. Yet, anyone who has ever driven a car has an intuitive grasp of calculus' most basic concepts: *differentiation* and *integration*. These two complementary operations may be seen at work on the instrument panel of every automobile:
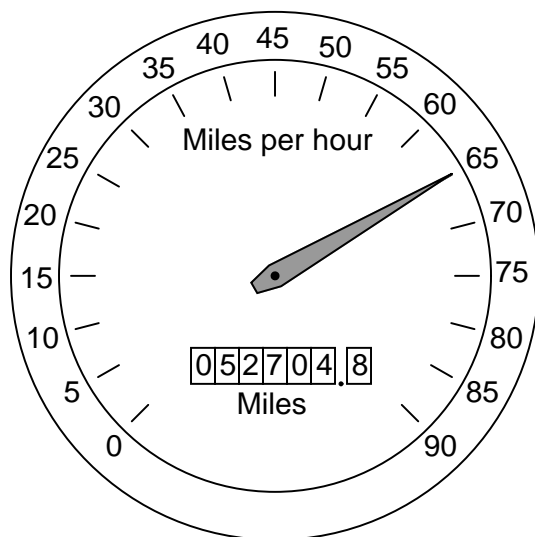
On this one instrument, two measurements are given: speed in miles per hour, and distance traveled in miles. In areas where metric units are used, the units would be kilometers per hour and kilometers, respectively. Regardless of units, the two variables of speed and distance are related to each other over time by the calculus operations of integration and differentiation. My question for you is which operation goes which way?

We know that speed is the rate of change of distance over time. This much is apparent simply by examining the units (miles *per hour* indicates a rate of change over time). Of these two variables, speed and distance, which is the *derivative* of the other, and which is the *integral* of the other? Also, determine what happens to the value of each one as the other maintains a constant (non-zero) value.

file 03648

Answer 67

Speed is the derivative of distance; distance is the integral of speed.

If the speed holds steady at some non-zero value, the distance will accumulate at a steady rate. If the distance holds steady, the speed indication will be zero because the car is at rest.
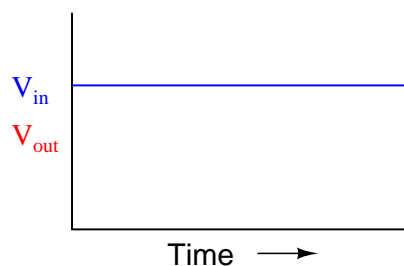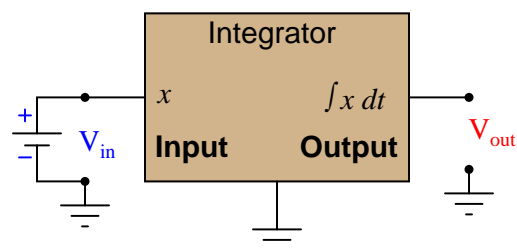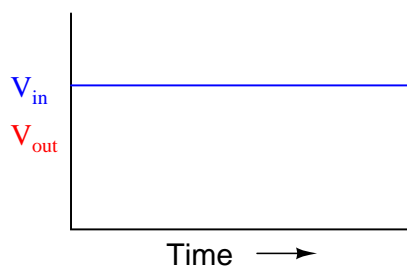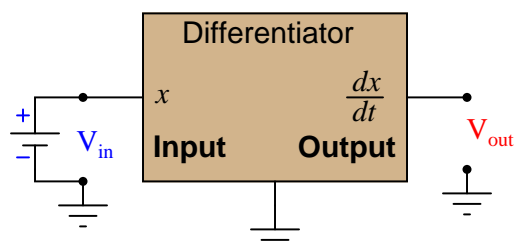
Notes 67

The goal of this question is to get students thinking in terms of derivative and integral every time they look at their car's speedometer/odometer, and ultimately to grasp the nature of these two calculus operations in terms they are already familiar with.
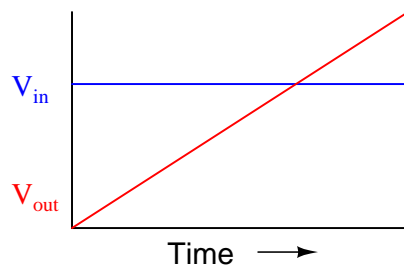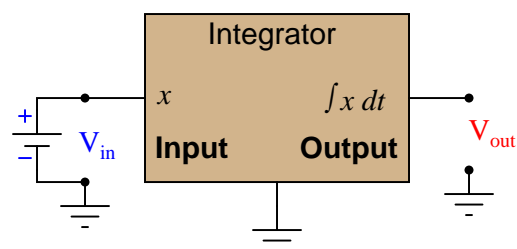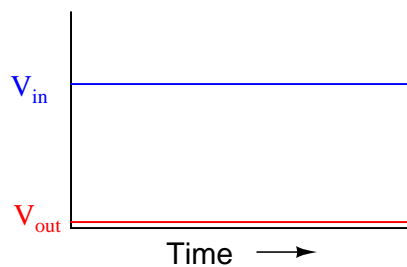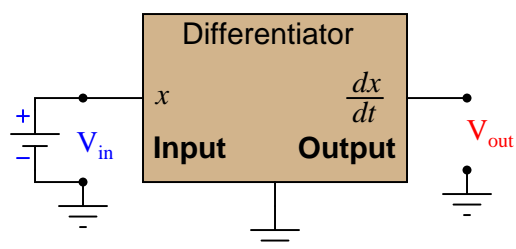
Question 68

$\int f(x)\,dx$ *Calculus alert!*

Determine what the response will be to a constant DC voltage applied at the input of these (ideal) circuits:

Answer 68

Notes 68

Ask your students to frame their answers in a practical context, such as speed and distance for a moving object (where speed is the time-derivative of distance and distance is the time-integral of speed).
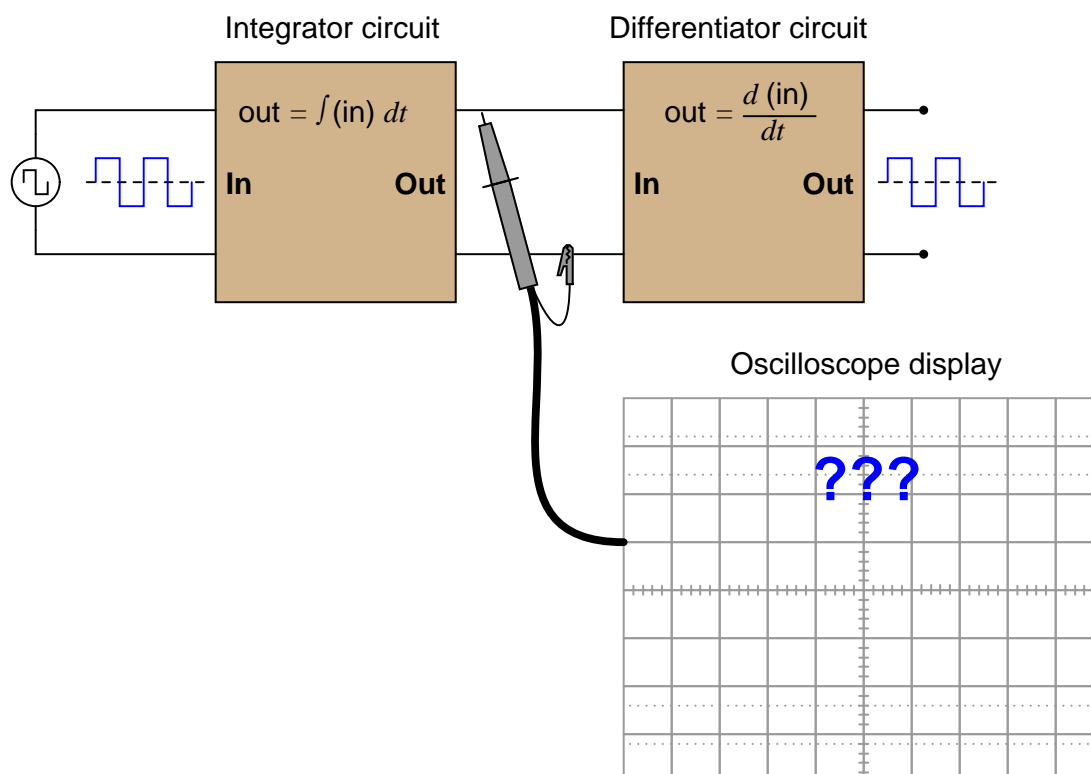
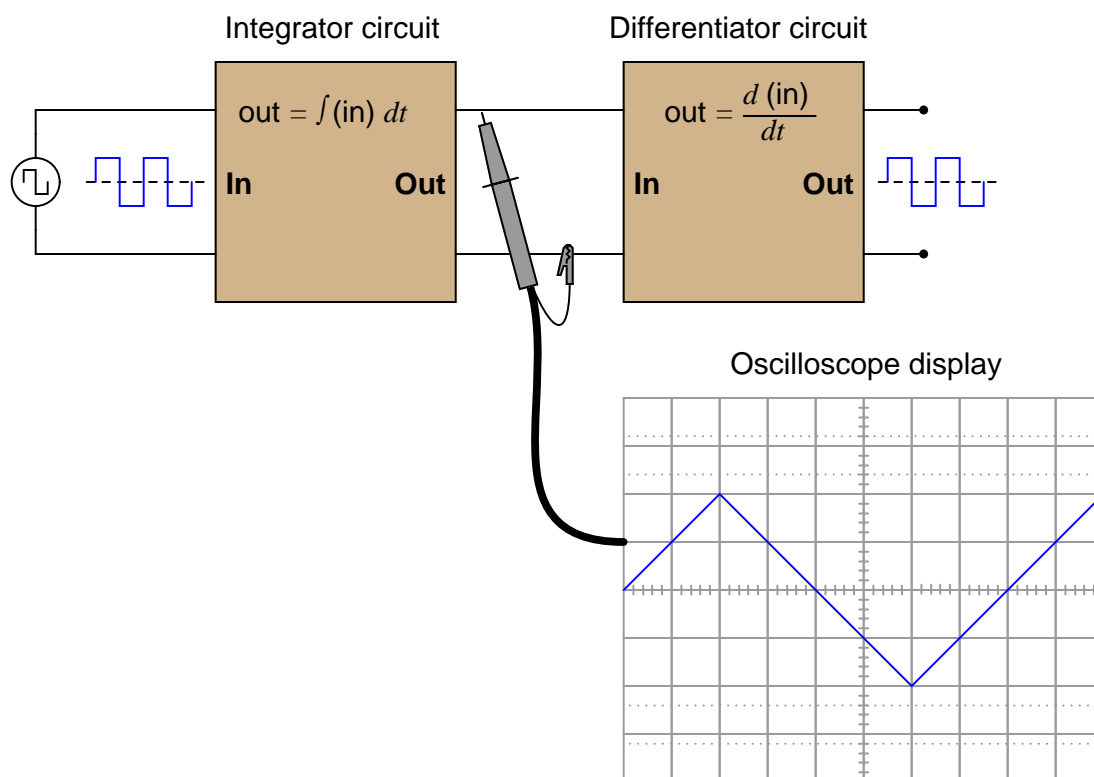$\boxed{\int f(x)\, dx \quad \textit{Calculus alert!}}$

In calculus, differentiation is the *inverse operation* of something else called *integration*. That is to say, differentiation "un-does" integration to arrive back at the original function (or signal). To illustrate this electronically, we may connect a differentiator circuit to the output of an integrator circuit and (ideally) get the exact same signal out that we put in:
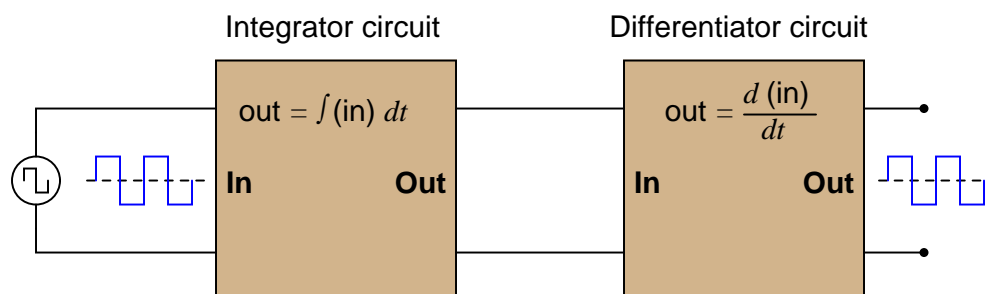
Based on what you know about differentiation and differentiator circuits, what must the signal look like in between the integrator and differentiator circuits to produce a final square-wave output? In other words, if we were to connect an oscilloscope in between these two circuits, what sort of signal would it show us?

file 03645

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Integrator circuit

Differentiator circuit

$$\text{out} = \int (\text{in})\ dt$$

In   **Out**

$$\text{out} = \frac{d\ (\text{in})}{dt}$$

In   **Out**

Oscilloscope display

Follow-up question: what do the schematic diagrams of passive integrator and differentiator circuits look like? How are they similar to one another and how do they differ?

Notes 69

This question introduces students to the concept of integration, following their prior familiarity with differentiation. Since they should already be familiar with other examples of inverse mathematical functions (arcfunctions in trigonometry, logs and powers, squares and roots, etc.), this should not be too much of a stretch. The fact that we may show them the cancellation of integration with differentiation should be proof enough.

In case you wish to demonstrate this principle "live" in the classroom, I suggest you bring a signal generator and oscilloscope to the class, and build the following circuit on a breadboard:
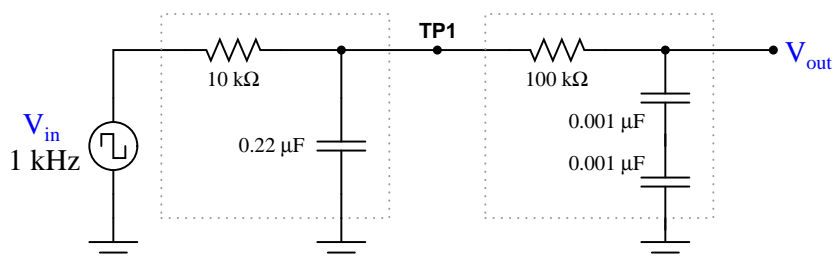

The output is not a perfect square wave, given the loading effects of the differentiator circuit on the integrator circuit, and also the imperfections of each operation (being passive rather than active integrator and differentiator circuits). However, the wave-shapes are clear enough to illustrate the basic concept.

Question 70

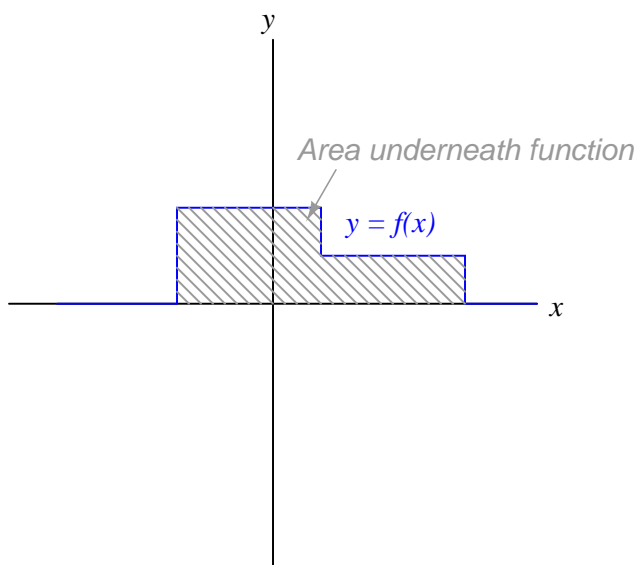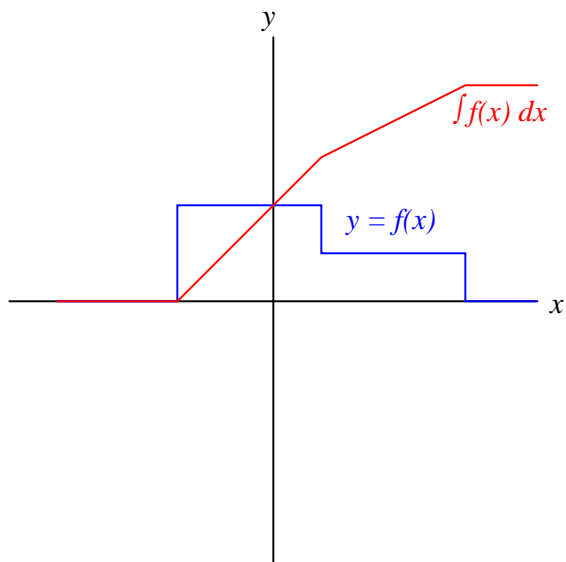$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Define what "integral" means when applied to the graph of a function. For instance, examine this graph:
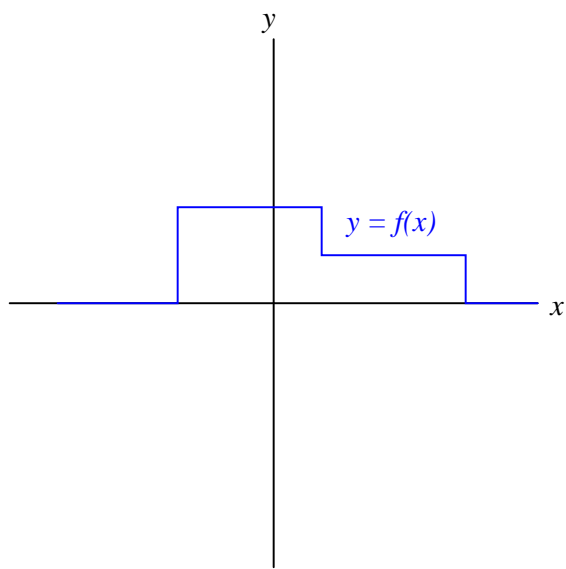
Sketch an approximate plot for the integral of this function.
file 03650

Answer 70

The graphical interpretation of "integral" means the *area* accumulated underneath the function for a given domain.

Usually students find the concept of the integral a bit harder to grasp than the concept of the derivative, even when interpreted in graphical form. One way to help them make this "leap" is to remind them that integration and differentiation are inverse functions, then ask them to analyze the answer "backwards" (looking at the red integral plot and seeing how the blue function is the derivative of the red function). The thought process is analogous to explaining logarithms to students for the very first time: when we take the logarithm of a number, we are figuring out what power we would have to raise the base to get that number (e.g. $\log 1000 = 3$; $10^3 = 1000$). When we determine the integral of a function, we are figuring out what other function, when differentiated, would result in the given function. This is the essence of what we mean by *inverse functions*, and it is an important concept in algebra, trigonometry, and calculus alike.
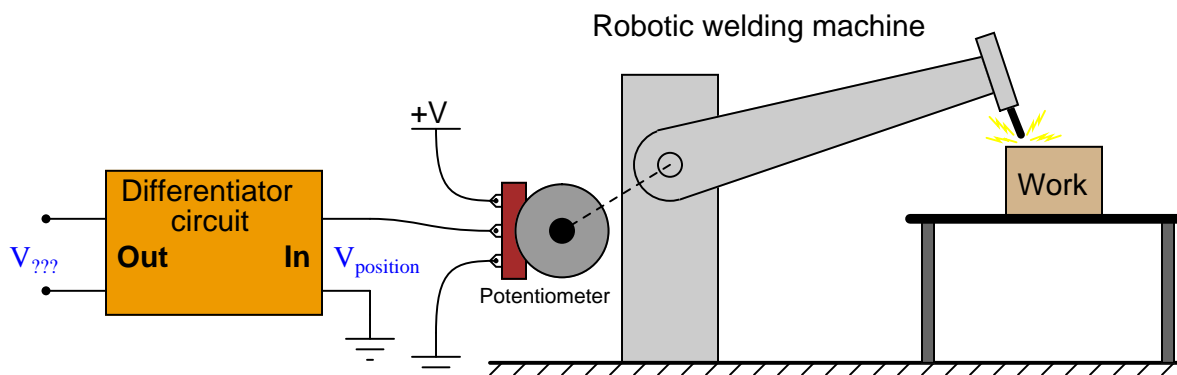
Question 71

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

Potentiometers are very useful devices in the field of robotics, because they allow us to represent the position of a machine part in terms of a voltage. In this particular case, a potentiometer mechanically linked to the joint of a robotic arm represents that arm's angular position by outputting a corresponding voltage signal:
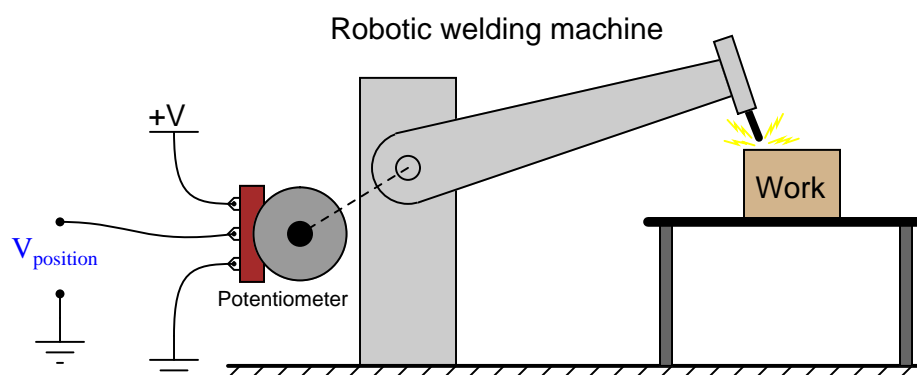
As the robotic arm rotates up and down, the potentiometer wire moves along the resistive strip inside, producing a voltage directly proportional to the arm's position. A voltmeter connected between the potentiometer wiper and ground will then indicate arm position. A computer with an analog input port connected to the same points will be able to measure, record, and (if also connected to the arm's motor drive circuits) control the arm's position.

If we connect the potentiometer's output to a *differentiator* circuit, we will obtain another signal representing something else about the robotic arm's action. What physical variable does the differentiator output signal represent?

file 03644

Answer 71

The differentiator circuit's output signal represents the angular *velocity* of the robotic arm, according to the following equation:

$$v = \frac{dx}{dt}$$

Where,
v = velocity
x = position
t = time

Follow-up question: what type of signal will we obtain if we differentiate the position signal twice (i.e. connect the output of the first differentiator circuit to the input of a second differentiator circuit)?

Notes 71

This question asks students to relate the concept of time-differentiation to physical motion, as well as giving them a very practical example of how a passive differentiator circuit could be used. In reality, one must be very careful to use differentiator circuits for real-world signals because differentiators tend to amplify high-frequency noise. Since real-world signals are often "noisy," this leads to a lot of noise in the differentiated signals.

Question 72

Logarithms have interesting properties, which we may exploit in electronic circuits to perform certain complex operations. In this question, I recommend you use a hand calculator to explore these properties.

Calculate the following:

- $10^{\log 3} =$
- $\log(10^8) =$

- $e^{\ln 3} =$
- $\ln(e^8) =$

- $10^{(\log 3 + \log 5)} =$
- $e^{(\ln 3 + \ln 5)} =$

- $10^{(\log 2.2 + \log 4)} =$
- $e^{(\ln 2.2 + \ln 4)} =$

- $10^{(\log 12 - \log 4)} =$
- $e^{(\ln 12 - \ln 4)} =$

- $10^{(2 \log 3)} =$
- $e^{(2 \ln 3)} =$

- $10^{\left(\frac{\log 25}{2}\right)} =$
- $e^{\left(\frac{\ln 25}{2}\right)} =$

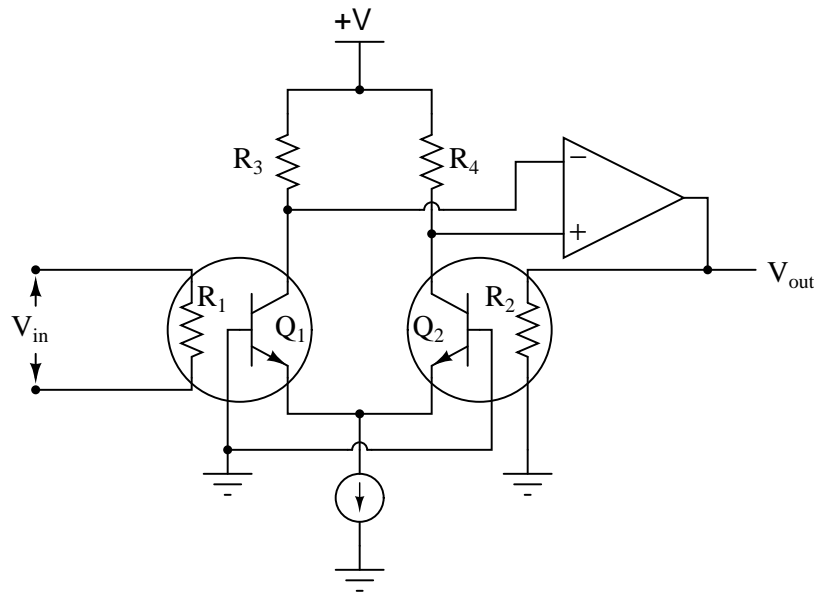<u>file 01018</u>

Answer 72

- $10^{\log 3} = 3$
- $\log(10^8) = 8$

- $e^{\ln 3} = 3$
- $\ln(e^8) = 8$

- $10^{(\log 3 + \log 5)} = 15$
- $e^{(\ln 3 + \ln 5)} = 15$

- $10^{(\log 2.2 + \log 4)} = 8.8$
- $e^{(\ln 2.2 + \ln 4)} = 8.8$

- $10^{(\log 12 - \log 4)} = 3$
- $e^{(\ln 12 - \ln 4)} = 3$

- $10^{(2 \log 3)} = 9$
- $e^{(2 \ln 3)} = 9$

- $10^{\left(\frac{\log 25}{2}\right)} = 5$
- $e^{\left(\frac{\ln 25}{2}\right)} = 5$

Discuss what mathematical operations are being done with the constants in these equations, by using logarithms. What patterns do your students notice? Also, discuss the terms "log" and "antilog," and relate them to opamp circuits they've seen.

Ask your students whether or not they think it matters what "base" of logarithm is used in these equations. Can they think of any other arithmetic operations to try using logarithms in this manner?

Question 73

In the early 1970's, the Fluke company invented a revolutionary new "RMS sensor" integrated circuit, used to convert an arbitrary waveform into its DC-equivalent (RMS) voltage. The device uses two precision resistors to heat a pair of matched transistors connected as a differential pair:

Describe how this circuit functions. What physical principle(s) does it use to derive an RMS value for V_{in}? Why is it important that all identical components (transistors, resistors) be precisely matched?

file 01014

Answer 73

This circuit exploits the temperature sensitivity of transistors to sense thermal balance between the two resistors R1 and R2. By definition, whatever DC voltage produces the same heat dissipation in a given resistance as an AC voltage is the RMS value of that AC voltage.

Notes 73

This question provides a good opportunity to review the function of differential pair circuits, and also the concept of RMS AC measurement. Ask your students how temperature influences the conductivity of bipolar junction transistors, and how the opamp's connection to resistor R2 forms a negative feedback loop.

Question 74

Write a mathematical expression in the form of $y = \ldots x$ describing the function of this circuit:
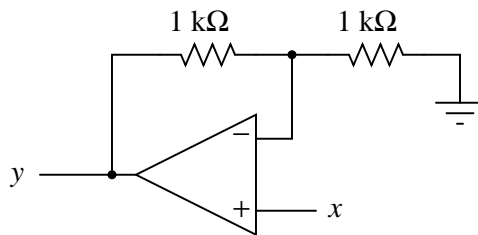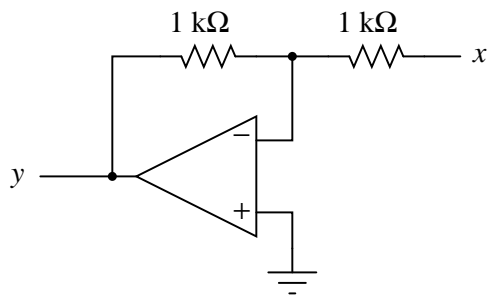

file 00999

Answer 74

$y = 2x$

Notes 74

Your students should be able to recognize that this amplifier circuit has a voltage gain of 2, but expressing it in the form of an equation using variables such as x and y may be something very new to them. Discuss with your students the significance of this notation: that a *circuit* may embody an *equation*. Analog computers may be "obsolete" technology, but they still have many practical applications.

Question 75

Write a mathematical expression in the form of $y = \ldots x$ describing the function of this circuit:
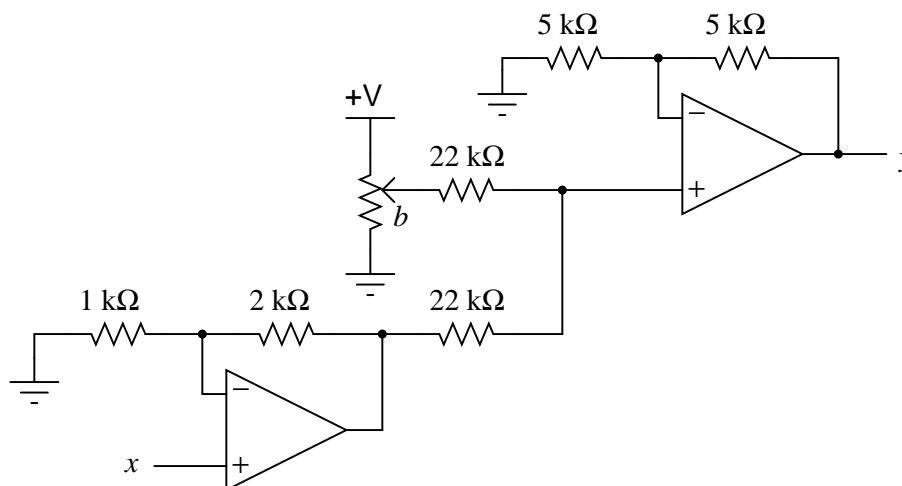
Answer 75

$y = -x$

Notes 75

Your students should be able to recognize that this amplifier circuit has a voltage gain of 1, and that it is *inverting* in nature, but expressing it in the form of an equation using variables such as x and y may be something very new to them. Discuss with your students the significance of this notation: that a *circuit* may embody an *equation*.

Question 76

Write a mathematical expression in the form of $y = \ldots x$ describing the function of this circuit:
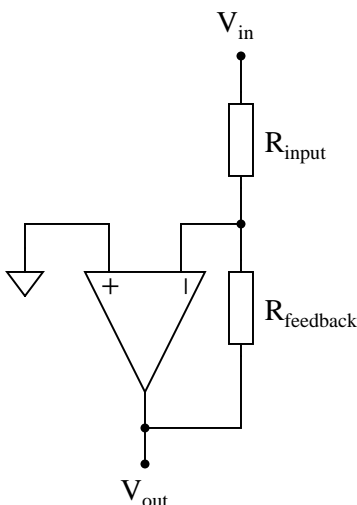
Answer 76

$y = 3x + b$

Notes 76

In this circuit, we combine two different op-amp functions into one system, embodying a linear equation. Discuss with your students what circuit alterations would be required to change the coefficients of this equation.

If it helps some of your students understand the overall function of this circuit better, you might want to ask other students to outline which groups of components in this circuit perform which portion of the equation (while the circuit schematic is displayed on the whiteboard for all to see).

Question 77

Ideally, an inverting amplifier circuit may be comprised of just one op-amp and two resistors, as such:

However, if high accuracy is desired, a third resistor must be added to the circuit, in series with the other op-amp input:
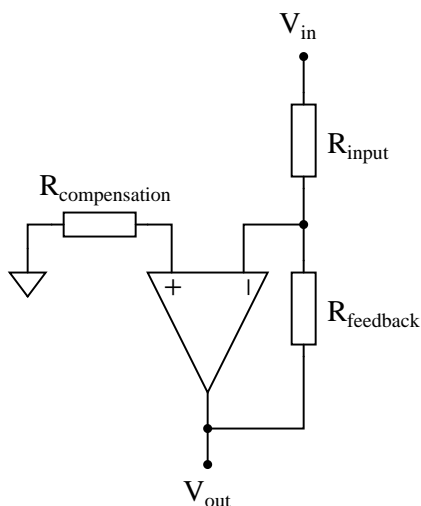
Explain what this "compensation" resistor is compensating for, and also what its value should be.
file 01007

Answer 77

The compensation resistor compensates for errors introduced into the voltage divider network due to input bias current. Its value should be equal to the parallel equivalent of R_{input} and $R_{feedback}$.

Notes 77

First, your students will have to know what "bias currents" are in op-amp circuits, so begin your discussion of this question with a call for this definition. Why the compensation resistor value must be equal to the *parallel* equivalent of the two resistors in the voltage divider is something that confuses most students. The key to understanding it is network analysis, in particular Thévenin's and Norton's theorems.

Question 78

$\int f(x)\,dx$ *Calculus alert!*

We know that the output of an integrator circuit is proportional to the time-integral of the input voltage:
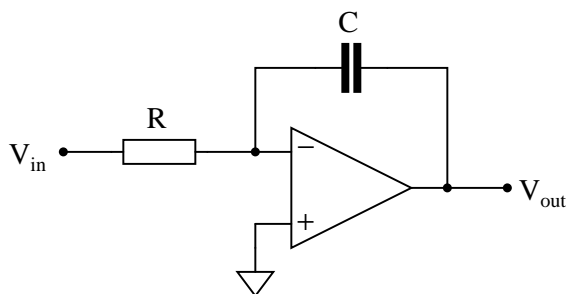
$$V_{out} \propto \int_0^T V_{in}\,dt$$

But how do we turn this proportionality into an exact equality, so that it accounts for the values of R and C? Although the answer to this question is easy enough to simply look up in an electronics reference book, it would be great to actually derive the exact equation from your knowledge of electronic component behaviors! Here are a couple of hints:

$$I = \frac{V}{R} \qquad\qquad i = C\frac{dv}{dt}$$

file 02704

Answer 78

$$V_{out} = -\frac{1}{RC} \int_0^T V_{in}\,dt$$

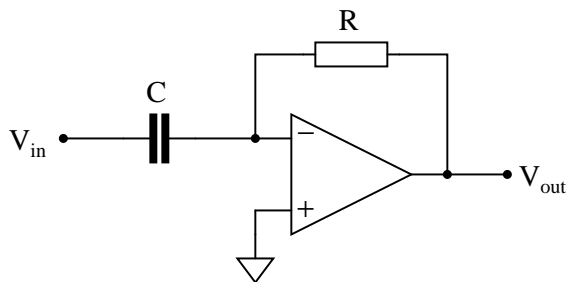Follow-up question: why is there a negative sign in the equation?

Notes 78

The two "hint" equations given at the end of the question beg for algebraic substitution, but students must be careful which variable(s) to substitute! Both equations contain an I, and both equations also contain a V. The answer to that question can only be found by looking at the schematic diagram: do the resistor and capacitor share the same current, the same voltage, or both?

Question 79

$\boxed{\int f(x)\,dx \quad \textit{Calculus alert!}}$

We know that the output of a differentiator circuit is proportional to the time-derivative of the input voltage:

$$V_{out} \propto \frac{dV_{in}}{dt}$$

But how do we turn this proportionality into an exact equality, so that it accounts for the values of R and C? Although the answer to this question is easy enough to simply look up in an electronics reference book, it would be great to actually derive the exact equation from your knowledge of electronic component behaviors! Here are a couple of hints:

$$I = \frac{V}{R} \qquad\qquad i = C\frac{dv}{dt}$$

file 02705

Answer 79

$$V_{out} = -RC\left(\frac{dV_{in}}{dt}\right)$$

Follow-up question: why is there a negative sign in the equation?

Notes 79

The two "hint" equations given at the end of the question beg for algebraic substitution, but students must be careful which variable(s) to substitute! Both equations contain an I, and both equations also contain a V. The answer to that question can only be found by looking at the schematic diagram: do the resistor and capacitor share the same current, the same voltage, or both?

Question 80

$\int f(x)\,dx$ *Calculus alert!*

A Rogowski coil has a mutual inductance rating of 5 μH. Calculate the size of the resistor necessary in the integrator circuit to give the integrator output a 1:1 scaling with the measured current, given a capacitor size of 4.7 nF:

Rogowski coil
(air-core current transformer)
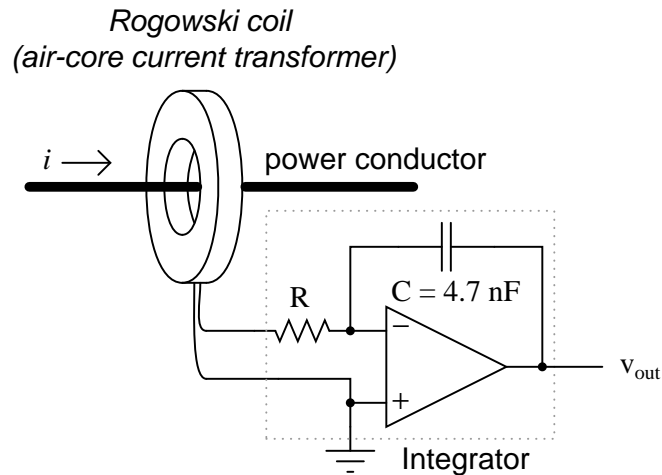
That is, size the resistor such that a current through the conductor changing at a rate of 1 amp per second will generate an integrator output voltage changing at a rate of 1 volt per second.

file 02703

Answer 80

$R = 1.064$ kΩ

Notes 80

This question not only tests students' comprehension of the Rogowski coil and its associated calculus (differentiating the power conductor current, as well as the need to integrate its output voltage signal), but it also tests students' quantitative comprehension of integrator circuit operation and problem-solving technique. Besides, it gives some practical context to integrator circuits!

Question 81

Practical integrator circuits must have a *compensating resistor* connected in parallel with the capacitor, in the feedback loop. Typically, this resistor value is very large: about 100 times as large as R_{in}.
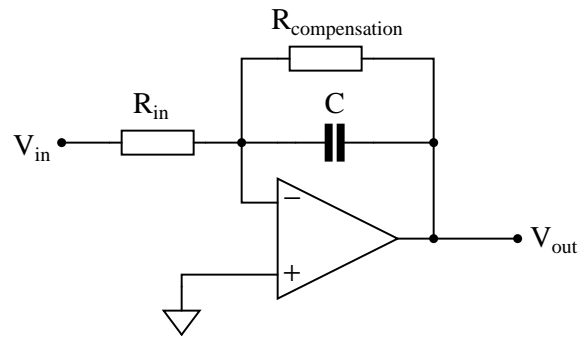
Describe why this is a necessity for accurate integration. Hint: an ideal opamp would not need this resistor!

file 01012

Answer 81

This compensation resistor helps offset errors otherwise incurred by the opamp's *bias current* on the inverting input.
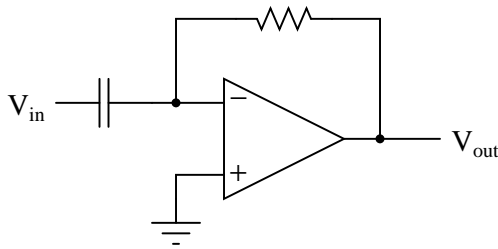
Notes 81

Discuss where bias currents originate from in the opamp's internal circuitry, and ask your students if they have any recommendations on specific opamp types that minimize bias current.
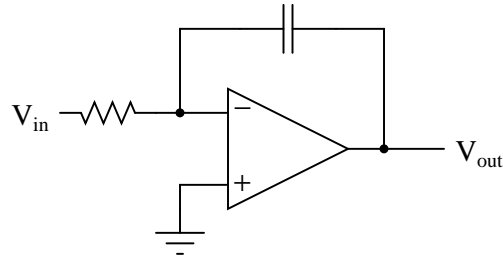
Question 82

Practical op-amp integrator and differentiator circuits often cannot be built as simply as their "textbook" forms usually appear:
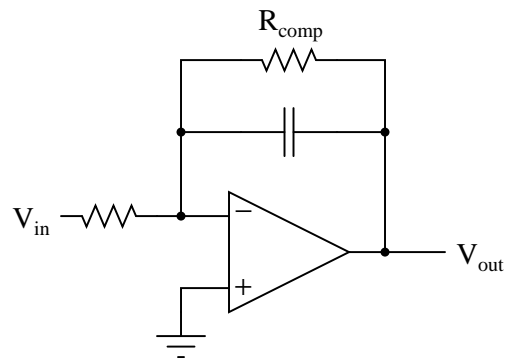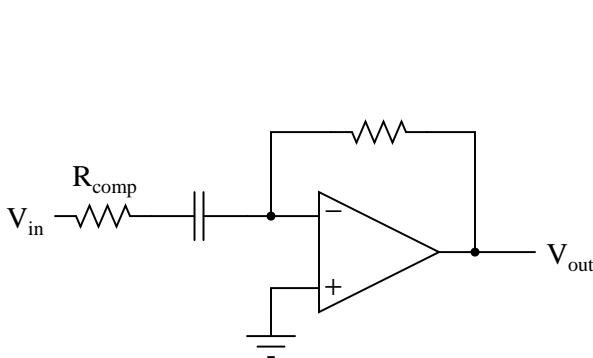
Ideal op-amp differentiator circuit   Ideal op-amp integrator circuit

The voltage gains of these circuits become extremely high at certain signal frequencies, and this may cause problems in real circuitry. A simple way to "tame" these high gains to moderate levels is to install an additional resistor in each of the circuits, as such:

The purpose of each resistor is to "dominate" the impedance of the RC network as the input signal frequency approaches the point at which problems would occur in the ideal versions of the circuits. In each of these "compensated" circuits, determine whether the value of the compensation resistor needs to be large or small compared to the other resistor, and explain why.

Of course, this solution is not without problems of its own. By adding this new resistor to each circuit, a half-power (-3 dB) cutoff frequency point is created by the interaction of the compensation resistor and the capacitor, as predicted by the equation $f_c = \frac{1}{2\pi R_{comp}C}$. The value predicted by this equation establishes a practical limit for the differentiation and integration functions, respectively. Operating on the wrong side of the frequency limit will result in an output waveform that is not the true time-derivative or time-integral of the input waveform. Determine whether the f_c value constitutes a *low* frequency limit or a *high* frequency limit for each circuit, and explain why.

file 01538

Answer 82

For the differentiator circuit, $R_{comp} << R_{feedback}$, and $\frac{1}{2\pi R_{comp}C}$ is a *high* frequency limit.

For the integrator circuit, $R_{comp} >> R_{input}$, and $\frac{1}{2\pi R_{comp}C}$ is a *low* frequency limit.

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In order to successfully answer the questions, students will need a firm grasp on the nature of the gain problem with differentiator and integrator circuits. This is based on an understanding of capacitive reactance, and the relationship between op-amp gain and input/feedback impedances. These are not difficult questions to answer, if one takes an orderly and methodical approach to the problem. Help your students reason through to the correct answers by asking questions that challenge them to link concepts of op-amp feedback gain, capacitive reactance, impedances of series and parallel RC networks, etc.

Question 83

Analog computers have been all but replaced by digital computers in modern electronic systems. Yet, analog computational circuits still enjoy certain advantages over digital circuits. Describe what some of the limitations of analog computers are, and why these limitations have led to their obsolescence. Also, discuss some of the advantages of analog computers, and why a designer might still choose to use an analog computational circuit in a modern system.

file 01005

Answer 83

Analog computational circuits are much less precise than their digital counterparts. On the other hand, analog circuits tend to be much simpler than digital circuits (for the same functions), and they are generally faster.

Notes 83

I like to introduce analog computational circuits to beginning electronics students because of their elegant simplicity, and for the fact that they greatly help to "link" the abstract world of mathematics to real mechanisms. Students are generally excited to realize they can build an actual *computer* with just a handful of inexpensive electronic components.