

Decibel measurements

This worksheet and all related files are licensed under the Creative Commons Attribution License, version 1.0. To view a copy of this license, visit <http://creativecommons.org/licenses/by/1.0/>, or send a letter to Creative Commons, 559 Nathan Abbott Way, Stanford, California 94305, USA. The terms and conditions of this license allow for free copying, distribution, and/or modification of all licensed works by the general public.

Resources and methods for learning about these subjects (list a few here, in preparation for your research):

Question 1

What does it mean to speak of the *gain* of a circuit? This term is very commonly used when describing amplifier circuits, but it may also be used to describe circuits containing nothing but passive components, and are thus incapable of amplifying.

What letter is used to symbolize gain in mathematical equations?

file 00672

Answer 1

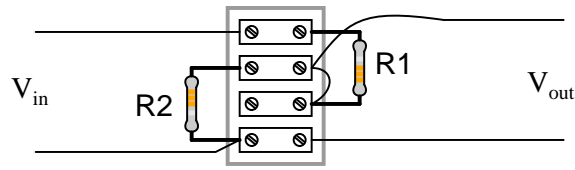
"Gain" (A) refers to the ratio of output signal compared to input signal.

Notes 1

Discuss what it means for a circuit to have a designated "output" and "input". Can they think of any circuits studied thus far that have places to input and output signals?

Question 2

Calculate the voltage gain of this circuit, if R1 has a resistance of 8.1 k Ω and R2 has a resistance of 1.75 k Ω :



file 00673

Answer 2

$$A_V = 0.178$$

Follow-up question: how does this gain figure (A_V) relate to the "voltage divider formula"?

$$E_R = E_{total} \left(\frac{R}{R_{total}} \right)$$

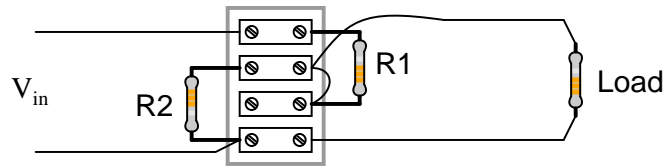
Notes 2

Students should readily recognize this circuit as a voltage divider, from their education in basic DC circuits. Though it may seem strange to calculate the "gain" of a completely passive and indeed *dissipative* circuit, it is entirely valid.

Discuss with your students the maximum and minimum possible power gain values for a circuit of this type.

Question 3

Calculate the power gain of this circuit, if R1 has a resistance of 1 k Ω , R2 has a resistance of 5.1 k Ω , and the load has a resistance of 10 k Ω :



[file 00674](#)

Answer 3

$$A_P = 0.261$$

Follow-up question: what *unit* does this figure have, if any?

Notes 3

Students should readily recognize this circuit from their education in basic DC circuits. Though it may seem strange to calculate the "gain" of a completely passive and indeed *dissipative* circuit, it is entirely valid.

Discuss with your students the maximum and minimum possible power gain values for a circuit of this type. Also discuss with them the nature of ratios with regard to units.

Question 4

During the early development of telephone technology, a unit was invented for representing power gain (or loss) in an electrical system. It was called the *Bel*, in honor of Alexander Graham Bell, the telecommunications pioneer.

"Bels" relate to power gain ratios by the following equation:

$$A_{P(ratio)} = 10^{A_{P(Bels)}}$$

Given this mathematical relationship, translate these power gain figures given in units of Bels, into ratios:

- $A_P = 3 \text{ B}$; $A_P =$
- $A_P = 2 \text{ B}$; $A_P =$
- $A_P = 1 \text{ B}$; $A_P =$
- $A_P = 0 \text{ B}$; $A_P =$
- $A_P = -1 \text{ B}$; $A_P =$
- $A_P = -2 \text{ B}$; $A_P =$
- $A_P = -3 \text{ B}$; $A_P =$

file 00675

Answer 4

- $A_P = 3 \text{ B}$; $A_P = 1000$
- $A_P = 2 \text{ B}$; $A_P = 100$
- $A_P = 1 \text{ B}$; $A_P = 10$
- $A_P = 0 \text{ B}$; $A_P = 1$
- $A_P = -1 \text{ B}$; $A_P = \frac{1}{10}$
- $A_P = -2 \text{ B}$; $A_P = \frac{1}{100}$
- $A_P = -3 \text{ B}$; $A_P = \frac{1}{1000}$

Follow-up question: a geologist, taking a class on electronics, sees this mathematical pattern and remarks, "This is just like the *Richter* scale!" Explain what the geologist means.

Notes 4

Ask your students how these two systems of power gain expression (Bels versus ratios) compare in terms of *range*. Which system of expression encompasses the greatest range of power gains or losses, with the smallest changes in numerical value?

Question 5

Manipulate this equation algebraically, so that we can convert power gains expressed in units of Bels, into ratios.

$$A_{P(ratio)} = 10^{A_{P(Bels)}}$$

Then, convert the following power gains, expressed as ratios, into units of Bels:

- $A_P = 250$; $A_P =$
- $A_P = 1275$; $A_P =$
- $A_P = 10$; $A_P =$
- $A_P = 1$; $A_P =$
- $A_P = 0.1$; $A_P =$
- $A_P = 0.025$; $A_P =$
- $A_P = 0.00009$; $A_P =$

file 00676

Answer 5

$$A_{P(Bels)} = \log A_{P(ratio)}$$

- $A_P = 250$; $A_P = 2.398$ B
- $A_P = 1275$; $A_P = 3.106$ B
- $A_P = 10$; $A_P = 1$ B
- $A_P = 1$; $A_P = 0$ B
- $A_P = 0.1$; $A_P = -1$ B
- $A_P = 0.025$; $A_P = -1.602$ B
- $A_P = 0.00009$; $A_P = -4.046$ B

Notes 5

Challenge your students to estimate the log values without using their calculators. For example, they should be able to estimate the log of 1275 as being between 3 and 4; the log of 0.025 as being between -1 and -2. Work together to devise a technique for doing this, where there will be no guessing.

Mathematical estimation is an important skill for technical people to possess. Not only is it useful in the event no calculator is readily available, but it also helps greatly in students being able check their (electronically) calculated work. I can't tell you how many times I've seen students blindly enter numbers into a calculator, only to arrive at an answer that is *grossly* in error, and not realize it at all because they cannot do the estimation mentally.

Question 6

At some point in time, it was decided that the unit of the "Bel" was too large. Instead, the *deci*-Bel became the most common usage of the unit. Modify these equations to include A_P figures cast in units of decibels (dB) instead of Bels:

$$A_{P(ratio)} = 10^{A_{P(Bels)}}$$

$$A_{P(Bels)} = \log A_{P(ratio)}$$

Then, calculate the decibel figures that correspond to a power gain of 2 (ratio), and a power loss of 50%, respectively.

file 00677

Answer 6

$$A_{P(ratio)} = 10^{\frac{A_{P(dB)}}{10}}$$

$$A_{P(dB)} = 10 \log A_{P(ratio)}$$

Power gain of 2 (ratio) \approx 3 dB

Power loss of 50% (ratio) \approx -3 dB

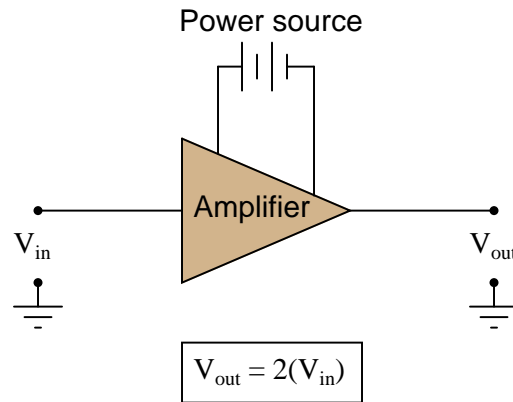
Notes 6

It is important that students work through the original equations algebraically to obtain the answers rather than just look up these formulae in a book. Have your students write their work on the whiteboard in front of the other students, so that everyone has the opportunity to examine the technique(s) and ask pertinent questions.

Be sure to let your students know that the figure of "3 dB", either positive or negative, is very common in electronics calculations. Your students might remember this expression used to describe the cutoff frequency of a filter circuit (f_{-3dB}).

Question 7

Suppose an AC signal amplifier circuit has a voltage gain (ratio) of 2. That is, V_{out} is twice as large as V_{in} :



If we were to try to rate this amplifier's gain in terms of the relative *power* dissipated by a given load resistance (P_{load} when powered by V_{out} , versus P_{load} when powered by V_{in}), what ratio would we calculate? In other words, what is the ratio of power for a given load resistance, when powered by a given voltage, versus when powered by a voltage that is twice as much?

[file 00826](#)

Answer 7

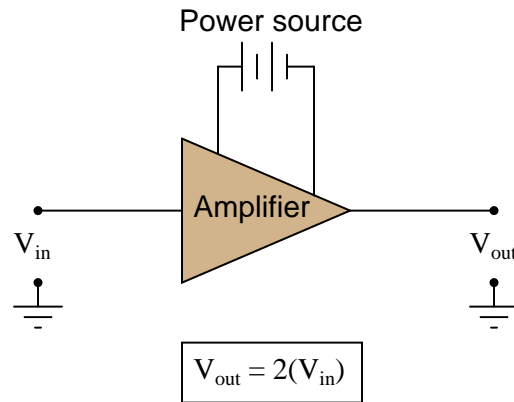
Power ratio = 4:1

Notes 7

An easy way to illustrate this principle is to ask your students to calculate the power dissipation of a 1200 watt heating element rated for 120 volts, if connected to a 240 volt source. The answer is *not* 2400 watts!

Question 8

Suppose an AC signal amplifier circuit has a voltage gain (ratio) of 2. That is, V_{out} is twice as large as V_{in} :



If we were to try to rate this amplifier's gain in terms of the relative power dissipated by a given load resistance (P_{load} when powered by V_{out} , versus P_{load} when powered by V_{in}), what *decibel* figure would we calculate?

file 00827

Answer 8

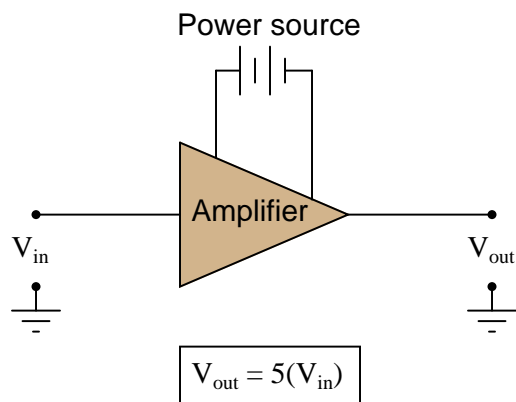
$$A_P = 6.02 \text{ dB}$$

Notes 8

An easy way to illustrate this principle is to ask your students to calculate the power dissipation of a 1200 watt heating element rated for 120 volts, if connected to a 240 volt source. The answer is *not* 2400 watts!

Question 9

Suppose an AC signal amplifier circuit has a voltage gain (ratio) of 5. That is, V_{out} is five times as large as V_{in} :



Translate this voltage gain ratio into a decibel figure. Explain why the conversion from voltage gain ratio to decibels is not the same as conversion of a power gain ratio to decibels.

file 00828

Answer 9

$$A_V = 13.98 \text{ dB}$$

Notes 9

Discuss with your students the nature of the "Bel" unit: it is fundamentally a unit of *power* gain, not voltage or current gain. So, representing voltage or current gains in units of either Bels or decibels means representing those voltage or current gains *in terms of how much power gain they equate to*.

Question 10

Voltage and current gains, expressed in units of decibels, may be calculated as such:

$$A_{V(dB)} = 10 \log (A_{V(ratio)})^2$$

$$A_{I(dB)} = 10 \log (A_{I(ratio)})^2$$

Another way of writing this equation is like this:

$$A_{V(dB)} = 20 \log A_{V(ratio)}$$

$$A_{I(dB)} = 20 \log A_{I(ratio)}$$

What law of algebra allows us to simplify a logarithmic equation in this manner?

file 00830

Answer 10

$$\log a^b = b \log a$$

Challenge question: knowing this algebraic law, solve for x in the following equation:

$$520 = 8^x$$

Notes 10

Logarithms are a confusing, but powerful, algebraic tool. In this example, we see how the logarithm of a power function is converted into a simple multiplication function.

The challenge question asks students to apply this relationship to an equation not containing logarithms at all. However, the fundamental rule of algebra is that you may perform any operation (including logarithms) to any equation so long as you apply it equally to *both sides* of the equation. Logarithms allow us to take an algebra problem such as this and simplify it significantly.

Question 11

Convert the following amplifier gains (either power, voltage, or current gain ratios) into gains expressed in the unit of decibels (dB):

- $A_P = 25$; $A_{P(dB)} =$
- $A_V = 10$; $A_{V(dB)} =$
- $A_I = 37$; $A_{I(dB)} =$
- $A_P = 150$; $A_{P(dB)} =$
- $A_I = 41$; $A_{I(dB)} =$
- $A_V = 3.4$; $A_{V(dB)} =$
- $A_P = 18$; $A_{P(dB)} =$
- $A_V = 100$; $A_{V(dB)} =$

file 02447

Answer 11

- $A_P = 25$; $A_{P(dB)} = 13.98$ dB
- $A_V = 10$; $A_{V(dB)} = 20$ dB
- $A_I = 37$; $A_{I(dB)} = 31.36$ dB
- $A_P = 150$; $A_{P(dB)} = 21.76$ dB
- $A_I = 41$; $A_{I(dB)} = 32.26$ dB
- $A_V = 3.4$; $A_{V(dB)} = 10.63$ dB
- $A_P = 18$; $A_{P(dB)} = 12.55$ dB
- $A_V = 100$; $A_{V(dB)} = 40$ dB

Notes 11

Nothing special here, just straightforward ratio-to-decibel calculations. Have your students share and discuss the steps necessary to do all these conversions.

Question 12

Convert the following amplifier gains expressed in the unit of decibels (dB), to gain figures expressed as unitless ratios:

- $A_P = 5 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 23 \text{ dB}$; $A_{V(ratio)} =$
- $A_I = 20 \text{ dB}$; $A_{I(ratio)} =$
- $A_P = 2.5 \text{ dB}$; $A_{P(ratio)} =$
- $A_I = 7.4 \text{ dB}$; $A_{I(ratio)} =$
- $A_V = 45 \text{ dB}$; $A_{V(ratio)} =$
- $A_P = 12.8 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 30 \text{ dB}$; $A_{V(ratio)} =$

file 02448

Answer 12

- $A_P = 5 \text{ dB}$; $A_{P(ratio)} = 3.16$
- $A_V = 23 \text{ dB}$; $A_{V(ratio)} = 14.13$
- $A_I = 20 \text{ dB}$; $A_{I(ratio)} = 10$
- $A_P = 2.5 \text{ dB}$; $A_{P(ratio)} = 1.78$
- $A_I = 7.4 \text{ dB}$; $A_{I(ratio)} = 2.34$
- $A_V = 45 \text{ dB}$; $A_{V(ratio)} = 177.8$
- $A_P = 12.8 \text{ dB}$; $A_{P(ratio)} = 19.05$
- $A_V = 30 \text{ dB}$; $A_{V(ratio)} = 31.62$

Notes 12

Nothing special here, just straightforward decibel-to-ratio calculations. Have your students share and discuss the steps necessary to do all these conversions.

Question 13

Convert the following amplifier gains between decibels and (unitless) ratios as necessary:

- $A_V = 14.1 \text{ dB}$; $A_{V(ratio)} =$
- $A_I = 202$; $A_{I(dB)} =$
- $A_P = 15 \text{ dB}$; $A_{P(ratio)} =$
- $A_I = 33$; $A_{I(dB)} =$
- $A_P = 49 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 57$; $A_{V(dB)} =$
- $A_P = 8.8 \text{ dB}$; $A_{P(ratio)} =$
- $A_V = 30$; $A_{V(dB)} =$

file 02449

Answer 13

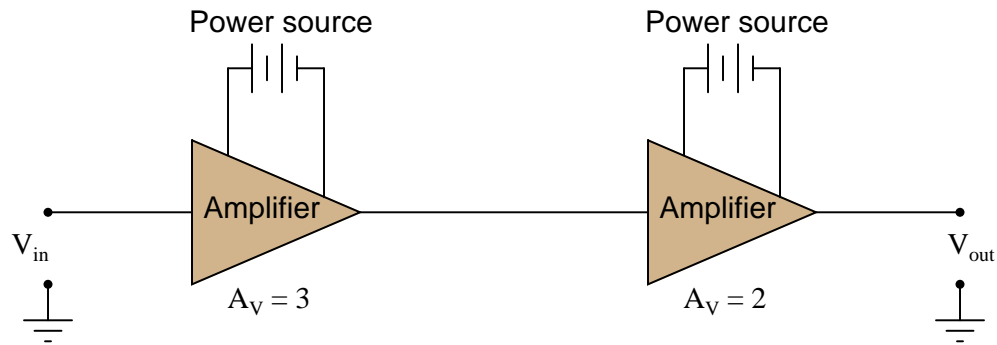
- $A_V = 14.1 \text{ dB}$; $A_{V(ratio)} = 5.07$
 - $A_I = 202$; $A_{I(dB)} = 46.1 \text{ dB}$
 - $A_P = 15 \text{ dB}$; $A_{P(ratio)} = 31.62$
 - $A_I = 33$; $A_{I(dB)} = 30.37 \text{ dB}$
 - $A_P = 49 \text{ dB}$; $A_{P(ratio)} = 79,432$
 - $A_V = 57$; $A_{V(dB)} = 35.12 \text{ dB}$
 - $A_P = 8.8 \text{ dB}$; $A_{P(ratio)} = 7.59$
 - $A_V = 30$; $A_{V(dB)} = 29.54 \text{ dB}$
-

Notes 13

Nothing special here, just straightforward decibel-ratio calculations. Have your students share and discuss the steps necessary to do all these conversions.

Question 14

Calculate the overall voltage gain of this cascaded amplifier circuit, where the output of one voltage amplifier feeds into the input of another:



Also, convert the voltage gains of each amplifier into units of decibels, then convert the overall voltage gain ratio into units of decibels as well.

What do you notice about the overall gain of this circuit in relation to the individual amplifier gains, compared as ratios versus compared as decibel figures?

[file 02534](#)

Answer 14

Cascaded voltage gains expressed as ratios:

$$A_V = 3 \times 2 = 6$$

Cascaded voltage gains expressed a decibel figures:

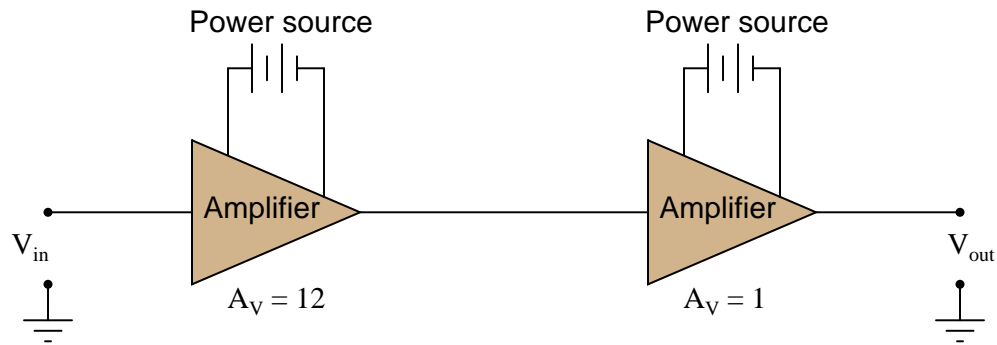
$$A_V = 9.54 \text{ dB} + 6.02 \text{ dB} = 15.56 \text{ dB}$$

Notes 14

Besides providing practice with ratio-to-decibel conversions, the purpose of this question is for students to realize that gains multiply as ratios but add as decibels.

Question 15

Calculate the overall voltage gain of this cascaded amplifier circuit, where the output of one voltage amplifier feeds into the input of another:



Also, convert the voltage gains of each amplifier into units of decibels, then convert the overall voltage gain ratio into units of decibels as well.

What do you notice about the overall gain of this circuit in relation to the individual amplifier gains, compared as ratios versus compared as decibel figures?

[file 02535](#)

Answer 15

Cascaded voltage gains expressed as ratios:

$$A_V = 12 \times 1 = 12$$

Cascaded voltage gains expressed a decibel figures:

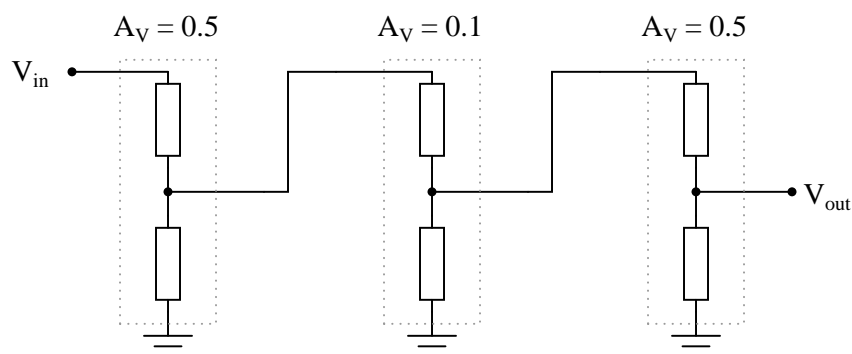
$$A_V = 21.58 \text{ dB} + 0 \text{ dB} = 21.58 \text{ dB}$$

Notes 15

Besides providing practice with ratio-to-decibel conversions, the purpose of this question is for students to realize that gains multiply as ratios but add as decibels.

Question 16

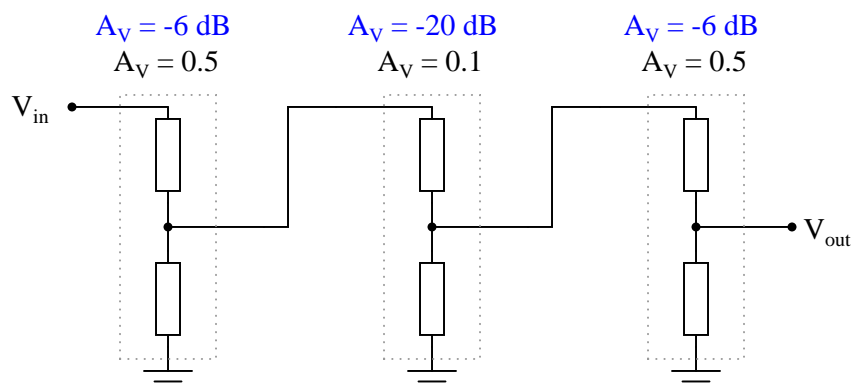
In this cascaded voltage divider circuit, determine the overall voltage gain ratio (from first input to last output), and also calculate the overall voltage gain in *decibels*, as well as the decibel figure for each divider's voltage gain:



What do you notice about the ratio figures versus the decibel figures, regarding how the individual stage gains compare with the overall gain?

[file 00829](#)

Answer 16



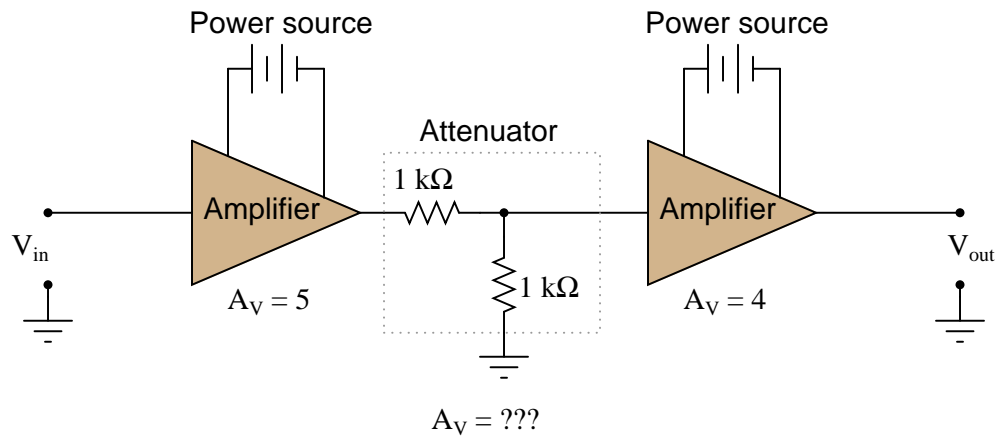
$$A_{V(\text{overall})} = -32 \text{ dB}$$
$$A_{V(\text{overall})} = 0.025 \text{ (ratio)}$$

Notes 16

Discuss with your students the nature of cascaded gain figures in both ratio and decibel formats. Which format is easier to calculate manually (without using a calculator)? Why is this?

Question 17

In this circuit, one amplifier feeds into an *attenuator* circuit, which then feeds into a second amplifier stage. Calculate the "gain" of the attenuator, and then calculate the overall voltage gain of this three-stage circuit:



Also, convert the voltage gains of each stage into units of decibels, then convert the overall voltage gain ratio into units of decibels as well.

What do you notice about the overall gain of this circuit in relation to the individual amplifier gains, compared as ratios versus compared as decibel figures?

[file 02536](#)

Answer 17

Cascaded voltage gains expressed as ratios:

$$A_V = 5 \times \left(\frac{1}{2}\right) \times 4 = 10$$

Cascaded voltage gains expressed a decibel figures:

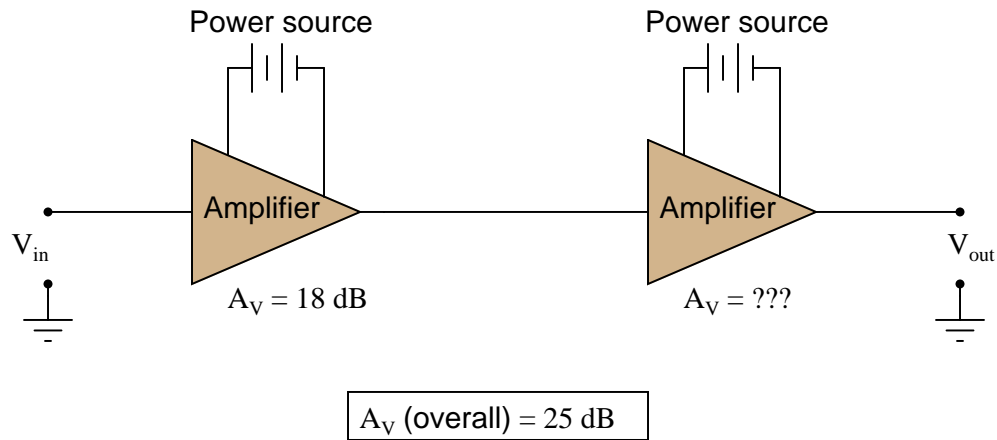
$$A_V = 13.98 \text{ dB} + (-6.02 \text{ dB}) + 12.04 = 20 \text{ dB}$$

Notes 17

Besides providing practice with ratio-to-decibel conversions, the purpose of this question is for students to realize that gains multiply as ratios but add as decibels.

Question 18

Calculate the necessary gain of the second-stage amplifier to give the whole circuit a voltage gain of 25 decibels, then translate all decibel figures into gain ratios:



file 02537

Answer 18

- Stage 1 gain = 18 dB = 7.94
- Stage 2 gain = 7 dB = 2.24
- Overall gain = 25 dB = 17.8

Notes 18

This question is nothing more than "drill" for students to practice decibel/ratio conversions.

Question 19

Specialized forms of the decibel unit have been devised to allow easy representation of quantities other than arbitrary ratios of voltage, current, or power. Take for example these units, the first one used extensively in the telecommunications industry:

- dBm
- dBW
- dBk

Define what each of these units represents.

file 00831

Answer 19

"dBm" represents the magnitude of a voltage in relation to 1 mW of power dissipated by a $600\ \Omega$ load. "dBW" and "dBk" units represent the magnitude of a voltage in relation to 1 W and 1 kW of power dissipated by the same load, respectively.

Follow-up question: how many volts is 2 dBm equivalent to?

Notes 19

Here we see the decibel unit being used to represent *absolute* quantities rather than relative ratios. Ask your students what benefit would there be in doing this. Why not just represent signal magnitudes in units of "volts" instead? Why would we want to use an obscure unit such as the decibel?